IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

SJPAS

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

Geometric Construction of a (57, 2)-Blocking Set in PG(2, 19) and Analysis of the $[324, 3, 306]_{19}$ Griesmer Code

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https://doi.org/10.54153/sjpas.2025.v7i1.1010

Article Information

Received: 04/11/2024
Revised: 10/01/2025
Accepted: 15/02/2025
Published: 30/03/2025

Keywords:

Arc, Bounded Griesmer , duble Blocking group, dropping $[n, k, d]_q$ code. projective techniques Plane, Optimal Linear code

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Abstract

In this paper, we explore the geometric structure of (57, 2)-blocking set in the projective plane PG(2, 19). By leveraging this structure, we construct a new (324, 18)-arc and derive a novel linear code with parameters $[\mathbf{324}, \mathbf{3}, \mathbf{306}]_{\mathbf{19}}$. Additionally, we systematically analyze the Grismer bound to determine whether this code is optimal or non-optimal, providing rigorous evidence through detailed stratification. Our investigation includes examples of arcs in the finite field PG(2, 19), and we demonstrate how these constructions contribute to the broader understanding of coding theory and finite geometry. The study also introduces new methodologies for identifying and characterizing blocking sets, arcs, and linear codes, expanding the potential for error correction, data transmission, and cryptography applications. By presenting concrete examples and theoretical insights, we aim to bridge the gap between geometric constructions and their practical implications in coding theory. Furthermore, this research underscores the significance of projective geometry in developing innovative solutions to long-standing problems in combinatorics and information theory. Through these findings, we contribute to the ongoing advancement of optimal code discovery and analysis within the finite field context.

Introduction:

The proportionality grade of the wheel X, at that point we will state that X is a wheel speaking to P(X). A sub space of measurement one is an arrangement of focuses the majority of whose speaking to wheels shape a sub space of measurement two of V (3, q).Such sub spaces are called streaks. The quantity of streaks in PG(2, q) is $q^2 + q$ +1. There are q + 1[5][6][1]

Example1:

We assume the code is linear and analyze the Grismuir bounds to determine whether it is optimal or non-optimal precisely $[325,3,307]_{19}$

Sol:- by best Grismer $\mathrm{nq(k, d)} \geq \mathrm{gq(k, d)} = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil$.

$n = nq(k, d) = \sum_{j=0}^{3-1} \left\lceil \frac{307}{19^j} \right\rceil$

$=\frac{307}{19^0}+\frac{307}{19^1}+\frac{307}{19^2}$

=307+16.1578947368+0.8504155125 $\cong$ 325 So that code is optimal.

Example2:

We assume the code is linear and analyze the Grismuir bounds to determine whether it is optimal or non-optimal precisely$[145,3,133]_{13}$.

Sol:- by best Grismer $nq(k,d) \geq gq(k,d) = \sum_{j=0}^{k-1}\left\lceil\frac{d}{q^j}\right\rceil$.

$n = nq(k,d) = \sum_{j=0}^{3-1}\left\lceil\frac{133}{13^j}\right\rceil$

$=\frac{133}{13^0}+\frac{133}{13^1}+\frac{133}{13^2}$

=133+10.23076923+0.786982248 $\cong$ 145

So that code is optimal**.**

Example3:

We assume the code is linear and analyze the Grismuir bounds to determine whether it is optimal or non-optimal precisely$[336,3,317]_{19}$.

Solution:- by best Grismer $\mathrm{nq(k,d)} \geq \mathrm{gq(k,d)} = \sum_{\mathrm{j=0}}^{\mathrm{k-1}}\left\lceil\frac{\mathrm{d}}{\mathrm{q^j}}\right\rceil$.

$\mathrm{n} = \mathrm{nq(k,d)} = \sum_{\mathrm{j=0}}^{3-1}\left\lceil\frac{317}{19^{\mathrm{j}}}\right\rceil$

$=\frac{317}{19^0}+\frac{317}{19^1}+\frac{317}{19^2}$

=317+16.68421053+0.87116343 $\leq$ 336

So that code is non-put Give GF(q) a chance to indicate the Galois area of q components and V (3, q) be the wheel space of column wheels of length three with sections in GF(q). Let PG(2, q) be the comparing projective techniques plane. The purposes of PG(2, q) are the non-zero wheels of V (3, q) with the standard that X =(x_1, x_2, x_3) and Y=(ƛx1,ƛx2,ƛx3) speak to a similar point, where ƛ ∈ GF(q)/{0}. The quantity of purposes of PG(2, q) is $q^2 + q$ +1.If the dot P(X) is [4]

Definition1:

A (k, r)- circular segment is an arrangement of k purposes of a projective techniques plane to such an extent that some r, however no r+1 of them, are collinear put [3]

Definition2:

A (l, n)- blocking group S in PG(2, q) is an arrangement of l focuses to such an extent that each line of PG(2, q) crosses S in at any rate n focuses, and there is a line meeting S inaccurately n focuses Note that a (k, r)- circular segment is the supplement of a ($q^2 + q$ +1-k, q + 1 − r)- blocking group in a projective techniques plane and alternately. Let V (n, q) signify the wheel space of all arranged n-tuples over GF(q). A direct code Cover GF(q) of length n and measurement k is a k-dimensional sub space of V (n, q). The wheels of C are called code words. The Hamming season between two code words is characterized to be the quantity of facilitate puts in which they differ. The least season of a code is the littlest of the season s between particular code words. Such a code is called a [n,k,d]_q-code if its minimum Hamming season is d. A focal issue in coding hypothesis is that of streamlining one of the parameters n, k and d for given estimations of the other two and q-settled. One of the variants is[2][9] [7]

Problem1:

Find nq(k, d), the littlest estimation of n for which there exists an$[n,k,d]_q$-code which accomplishes this esteem is called ideal. The outstanding slash destined for the capacity nq(k, d) is the accompanying Griesmer restricted[8]

$\mathrm{nq}(\mathrm{k},\mathrm{d}) \geq \mathrm{gq}(\mathrm{k},\mathrm{d}) = \sum_{\mathrm{j}=0}^{\mathrm{k}-1} \left\lceil \frac{\mathrm{d}}{\mathrm{q}^{\mathrm{j}}} \right\rceil$ and so are optimal.

Now give some examples using Griesmer base to see the optimal code and the non-optimal code

Codes with specific parameters are referred to as Griesmer codes. There exists a relationship between (n,r)(n, r)-circular segments in PG(2,9)PG(2, 9) and [n,3,d][n, 3, d]-codes, as described by the following hypothesis: [gq(k,d)]q[g_q(k, d)]_q.

Theroem1 :

There exists a projective techniques $[n,3,d]_q$code if and only if there exists an (n, n−d)-arc in PG(2, q). In this paper we consider the c ase q =19 and the elements of GF(19) are denoted by0,1,2,3, 4,5,6,7, 8, 9,10 ,11,12,13,14,15,16,17,18.

Theroem2:

There exists a (57, 2)- blocking group in PG(2, 19) and a (324,18)- arc in PG(2,19)

Table1: The straight line resulting from the intersection of the six points (1,1,4)...(1,15,15)

L_{17}

i	1	2	3	4	5	6	7	8	9	10
Mi	(1,1,4)	(1,12,14)	(1,14,2)	(0,1,13)	(1,9,13)	(1, 2,17)	(1,18,16)	(1,7,6)	(1,3,11)	(1,15,15)
i	11	12	13	14	15	16	17	18	19	20
Mi	(1,16,9)	(1,10,7)	(1,8,0)	(1,5,18)	(1,13,8)	(1,17,3)	(1,0,10)	(1,11,1)	(1,6,12)	(1,4,5)

Table2: The straight line resulting from the intersection of the six points (1,17,0)...(1,18,17)

L_{24}

i	1	2	3	4	5	6	7	8	9	10
Qi	(1,17,0)	(0,1,17)	(1,15,4)	(1,5,5)	(1,11,12)	(1,3,9)	(1,2,11)	(1,1,13)	(1,7,1)	(1,18,17)
i	11	12	13	14	15	16	17	18	19	20
Qi	(1,10,14)	(1,14,6)	(1,0,15)	(1,13,8)	(1,6,3)	(1,9,16)	(1,7,3)	(1,4,7)	(1,8,18)	(1,12,10)

Table3: The straight line resulting from the intersection of the six points (1,12,14)...(1,7,12)

L_{18}

i	1	2	3	4	5	6	7	8	9	10
Ni	(1,12,14)	(1,17,16)	(1,5,15)	(1,0,13)	(1,11,6)	(1,14,11)	(1,3,18)	(1,8,1)	(1,13,3)	(1,7,12)
i	11	12	13	14	15	16	17	18	19	20
Ni	(1,18,5)	(1,15,0)	(0,1,8)	(1,9,9)	(1,6,4)	(1,16,8)	(1,1,2)	(1,10,17)	(1,4,7)	(1,2,10)

Table4: The straight line resulting from the intersection of the six points (1,1,14)...(1,4,9)

L_{23}

i	1	2	3	4	5	6	7	8	9	10
Pi	(1,1,14)	(1,17,0)	(1,9,7)	(1,12,2)	(1,13,13)	(1,15,16)	(1,14,5)	(1,11,10)	(1,8,15)	(1,4,9)
i	11	12	13	14	15	16	17	18	19	20
Pi	(1,5,1)	(1,3,17)	(0,1,11)	(1,18,11)	(1,16,8)	(1,10,18)	(1,11,15)	(1,6,12)	(1,2,6)	(1,7,4)

he streaks li: aix +biy+ ciz=0, (i=1, 2, 3, 4) are chosen with the goal that each lineli contains the point (ai, bi, ci), (i=1, 2, 3, 4).The focuses Mi (I = 1, 2, . . . , 20) have a place with the stripe l17:x+9y+13z=0, The focuses Ni (I = 1, 2, . . . , 20) have a place with the linel18:x+11y+6z+0. The focuses Pi (I = 1, 2, . . . , 20) lie on hold

l23:x+13y+13z=0, and the focuses Qi (I =1, 2, . . . , 20) are the purposes of the linel24:x+14y+6z=0. The four lines meet pairwise at the focuses M1 =Q1,M2 = P2, N1 = P1, N2 = Q2, M6 = N6 and P11 = Q11, i.e. they are lines by and large position

$P1: Y = 0$	$P2: X = 0$
$p3: x + y = 0$	$p4: x + 2y = 0$
$p5: x + 3y = 0$	$p6: x + 4y = 0$
$p7: x + 5y = 0$	$p8: x + 6y = 0$
$p9: y + 7y = 0$	$p10: x + 8y = 0$
$p11: x + 9y = 0$	$p12: x + 10y = 0$
$p13: x + 11y = 0$	$p14: x + 12y = 0$
$p15: x + 13y = 0$	$p16: x + 14y = 0$
$p17: y + 15y = 0$	$p18: x + 16y = 0$
$p19: x + 17y = 0$	$p20: x + 18y = 0$

through the crossing point focuses M_2,M_{15}, M_{19}, N_{16},M_{19} and Q_1 .

The situation (a)– (c) will include that including the focuses A1, A2, A3,A4 to the arranging of wonderful goals of the streaks ,we will obtain a 2-blocking group without any than 56. Clearly we ought not evict any focuses from the four folds Mi , Ni , Pi , Qi , I = 1, 2; generally, the streaks p2: x = 0 and p1: y = 0 will shift across suitable 1-or 0-secants. Corres pondingly, it isn't alluringly to expelled any focused on from the four folds Mi , Ni , Pi , Qi , I = 6, 11, on the base that expelling a crossing point

The watchful investigation of the streaks L17, L18, L23 and L24 demonstrates that each fourfold (on account of I = 6, 11—each triple, and on account of I = 1, 2—each match) of focuses Mi , Ni , Pi , Qi (I = 1, 2 . . . , 20) has a place with one of the20 streaks pi. Presently given us a chance to group the accompanying undertaking: Remove22 focuses from the group L17∪L18∪ L23∪ L24, so that:

(a) There is no line in PG(2, 19) which is unique in relation to li and which contains four of the expelled focuses

(b) The streaks that contain three of the evacuated focuses meet at most four new focuses A1, A2, A3,

(c) The streaks that contain only two of the evacuated focuses don't go

purpose of the streaks liwill either not diminish enough the quantity of focuses, or the streaks p5: x + 3y = 0 or p12: x + 10y = 0 will move toward becoming 1-or 0-secants.We require toobserve the accompanying guideline: on the off chance that we have officially expelled two points from a fourfold Mi , Ni , Pi , Qi , we ought to leavethe staying two in the group. Else, one of the streaks pi will turn into a 1-or 0-secant.Let us take out the accompanying 22: from the lineL_{17} : 53, 68,109, 250,260,277, from the lineL18– 19, 73, 337, 16,251,261,278, from the lineL23– 23, 115, 283, 242 and from the lineL24–25, 47, 22

Now we select four streaks intersecting six points and streaks are L_{17},L_{18},L_{23},L_{24} such that

$L_{17} \cap L_{18} = 18 = (1,12,14)$

$L_{17} \cap L_{23} = 356 = (1,6,12)$

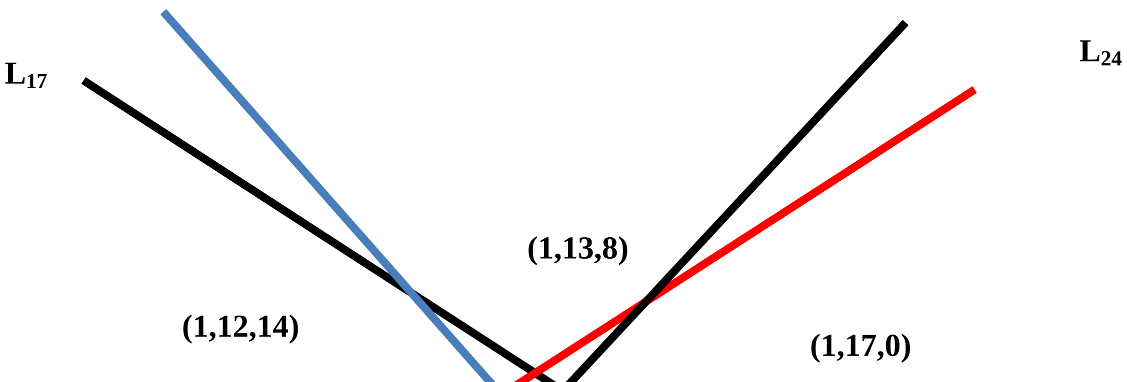
$L_{17} \cap L_{24} = 284 = (1,13,8)$

$L_{18} \cap L_{23} = 290 = (1,10,14)$

$L_{18} \cap L_{24} = 357 = (1,6,8)$

$L_{23} \cap L_{24} = 24 = (1,17,0)$

So the six common points are the sequence points [18,356,284,290,357,24]



It is anything but difficult to check now that none of them contains a crossing point purpose of streaks li; therefore condition (c) is fulfilled .Let us take a gander at condition (b). The 3-secants of An, i.e. the streaks not the same as li , with the end goal that each contains three of the evacuated focuses, are

$g_1 = 0x + y + 13z = 0$, $M_{11}, Q_{18}, P_4 \in g_1$

$g_2 = x + 9y + 13z = 0$, $M_9, N_{11}, P_{13} \in g_2$

$g_3 = x + 7y + 6z = 0$, $M_{13}, Q_6, N_5 \in g_3$

$g_4 = x + 16y + 9z = 0$, $M_4, N_{15}, P_{16} \in g_4$

$g_5 = x + 10y + 7z = 0$, $Q_1, P_2, N_9 \in g_5$

$g_6 = x + 5y + 18z = 0$, $Q_7, P_{18}, M_{19} \in g_6$

$g_7 = x + 4y + 5z = 0$, $Q_{13}, P_8, N_8 \in g_7$

$g_8 = x + 17y + 16z = 0$, $M_9, N_4, P_3 \in g_8$

$g_9 = x + 14y + 11z = 0$, $N_1, M_2, P_{19} \in g_9$

$g_{10} = x + 18y + 5z = 0$, $P_{14}, Q_{13}, N_{18} \in g_{10}$

$g_{11} = x + 15y + 0z = 0$, $Q_4, P_{11}, N_3 \in g_{11}$

$g_{12} = x + 9y + 9z = 0$, $M_{15}, N_7, P_8 \in g_{12}$

$g_{13} = x + y + 2z = 0$, $Q_{18}, P_{12}, N_{19} \in g_{13}$

$g_{14} = x + 2y + 10z = 0$, $M_{19}, N_9, P_{18} \in g_{14}$

$g_{15} = x + y + 14z = 0$, $M_2, N_1, P_8 \in g_{15}$

$g_{16} = x + 11y + 10z = 0$, $Q_7, P_1, M_{20} \in g_{16}$

$g_{17} = x + 18y + 11z = 0$, $M_{15}, N_{11}, P_{20} \in g_{17}$

$g_{18} = x + 11y + 15z = 0$, $M_{19}, N_7, Q_2 \in g_{18}$

$g_{19} = 0x + y + 17z = 0$, $Q_{15}, P_{17}, N_{16} \in g_{19}$

$g_{20} = x + 15y + 4z = 0$, $Q_{14}, P_{12}, N_5 \in g_{20}$

$g_{21} = x + 12y + 10z = 0$, $M_2, N_1, P_9 \in g_{21}$

Each line giintersects some line $\boldsymbol{li}$ at a point not in the group A. Indeed:

$g_1 \cap l_2 = (1,14,11)$, $g_2 \cap l_1 = (1,10,7)$

$g_3 \cap l_3 = (1,11,10)$, $g_4 \cap l_4 = (1,1,13)$

$g_5 \cap l_2 = (1,17,16)$, $g_6 \cap l_1 = (1,9,13)$

$g_7 \cap l_4 = (0,1,17)$, $g_8 \cap l_3 = (1,2,6)$

$g_9 \cap l_1 = (0,1,13)$, $g_{10} \cap l_4 = (1,12,10)$

$g_{10} \cap l_1 = (1{,}4{,}5)$, $g_{12} \cap l_1 = (1{,}7{,}9)$

$g_{11} \cap l_4 = (1{,}15{,}4)$, $g_{14} \cap l_2 = (1{,}0{,}13)$

$g_{12} \cap l_2 = (1{,}6{,}11)$, $g_{16} \cap l_4 = (1{,}13{,}6)$

$g_{13} \cap l_2 = (1{,}2{,}5)$, $g_{18} \cap l_4 = (1{,}9{,}16)$

$g_{14} \cap l_3 = (1{,}9{,}9)$, $g_{20} \cap l_2 = (1{,}3{,}18)$

$g_{21} \cap l_2 = (1{,}18{,}6)$

Furthermore, the streaks giintersect one another in four folds at the points More precisely

$g_2 \cap g_4 \cap g_6 \cap g_8 \cap g_{14} \cap g_{18} \cap g_{21} = (\mathbf{1}, \mathbf{12}, \mathbf{10})$

$g_7 \cap g_9 \cap g_{12} \cap g_{13} \cap g_{14} \cap g_{20} \cap g_{21} = (\mathbf{1}, \mathbf{11}, \mathbf{15})$

$g_1 \cap g_{13} \cap g_{22} = (\mathbf{1}, \mathbf{12}, \mathbf{16})$

$g_1 \cap g_2 \cap g_{14} \cap g_5 = (\mathbf{1}, \mathbf{7}, \mathbf{17})$

Therefore, (1,12,10), (1,11,15), (1,12,16), (1,7,17) are the points A_1, A_2, A_3, A_4.

Adding these four points to the rest 52 points, we obtain the group

$$B = \left\{ \begin{array}{c} (1{,}12{,}14), \\ (1{,}14{,}2), (1{,}2{,}17), (1{,}18{,}16), (1{,}3{,}11), (1{,}15{,}15), (1{,}8{,}0), \\ (1{,}17{,}3), (1{,}0{,}10) \\ (1{,}11{,}1) \\ (1{,}5{,}15) \\ (1{,}0{,}13), (1{,}11{,}6), (1{,}3{,}18), (1{,}8{,}1), (1{,}13{,}3), (1{,}7{,}12), (0{,}1{,}8), \\ (1{,}6{,}4), (1{,}10{,}17) \\ (1{,}9{,}7), (1{,}12{,}2), (1{,}13{,}13), (1{,}15{,}16), (1{,}14{,}5), (1{,}8{,}15), \\ (1{,}4{,}9), (1{,}5{,}1), (1{,}3{,}17), \\ (0{,}1{,}11), (1{,}1{,}4) \\ (1{,}10{,}18) \\ (1{,}11{,}12) \\ (1{,}6{,}2)(1{,}7{,}4)(1{,}5{,}5)(1{,}3{,}9)(1{,}2{,}11)(1{,}1{,}13)(1{,}7{,}1) \\ (1{,}18{,}17)(1{,}10{,}14)(1{,}14{,}6)(1{,}0{,}15)(1{,}6{,}3)(1{,}9{,}16)(1{,}7{,}3) \\ (1{,}8{,}18)(1{,}13{,}8)(1{,}6{,}12) \\ (1{,}16{,}8)(1{,}4{,}7)(1{,}17{,}0) \\ (1{,}12{,}16)(1{,}12{,}10)(1{,}7{,}17)(1{,}11{,}15) \end{array} \right\}$$

which is a (57, 2)- blocking group in PG(2, 19) and has the accompanying secant appropriation:

$$\tau_1 = 20\,, \tau_2 = 123, \tau_3 = 139, \tau_4 = 79, \tau_5 = 16\,, \tau_{13} = 2\,\,, \tau_{16} = 1, \tau_{17=1}$$

The supplement of the group B is a (324, 18)- bend. It pursues now by Theorem 1.7 that there exis $[324{,}3{,}306]_{19}$

Table5: Griesmer code

(324,18)-arc									
1	(1,0,0)	29	(1,14,15)	57	(0,1,5)	85	(1,17,2)	113	(1,8,9)
2	(0,1,0)	30	(1,7,5)	58	(1,0,4)	86	(1,15,11)	114	(1,18,14)
3	(0,0,1)	31	(1,2,16)	144	(1,16,1)	87	(1,13,10)	115	(1,11,10)
4	(1,0,2)	32	(1,3,8)	145	(1,10,10)	88	(1,1,15)	141	(1,3,16)
5	(1,5,2)	33	(1,6,1)	61	(1,2,12)	89	(1,7,9)	117	(1,11,13)
6	(1,5,8)	34	(1,10,5)	62	(1,4,10)	90	(1,18,14)	118	(1,11,9)
7	(1,6,13)	35	(1,2,3)	63	(1,1,6)	91	(1,17,4)	119	(1,18,10)
8	(1,11,11)	36	(1,16,15)	64	(1,8,10)	92	(1,12,16)	120	(1,1,1)
9	(1,13,12)	37	(1,7,0)	65	(1,1,10)	93	(1,3,0)	121	(1,10,12)
10	(1,4,16)	38	(0,1,7)	66	(1,1,3)	94	(0,1,3)	122	(1,4,4)
11	(1,3,14)	39	(1,0,17)	67	(1,16,18)	95	(1,0,18)	123	(1,12,12)
12	(1,17,15)	146	(1,1,12)	154	(1,16,7)	96	(1,9,2)	124	(1,4,12)
13	(1,7,7)	147	(1,4,6)	155	(1,15,14)	97	(1,5,9)	125	(1,4,18)
14	(1,15,12)	42	(1,7,18)	70	(1,8,14)	156	(1,17,10)	126	(1,9,0)
165	(1,15,9)	43	(1,9,8)	71	(1,17,5)	157	(1,1,0)	127	(0,1,9)
16	(1,2,10)	44	(1,6,18)	168	(1,11,14)	100	(1,9,10)	128	(1,0,1)
260	(1,10, 7)	45	(1,9,18)	73	(1,14,11)	101	(1,1,11)	129	(1,10,2)
166	(1,18,6)	167	(1,8,13)	169	(1,17,18)	102	(1,13,15)	130	(1,5,14)
19	(1,17,16)	47	(1,15,4)	53	(0,1,13)	103	(1,7,17)	131	(1,17,11)
20	(1,3,15)	48	(1,12,11)	76	(1,4,8)	162	(1,16,6)	132	(1,13,14)
150	(1,10,15)	49	(1,13,6)	77	(1,6,7)	163	(1,8,16)	133	(1,17,14)

151	(1,7,15)	50	(1,8,11)	160	(1,4,2)	106	(1,13,11)	134	(1,17,6)
152	(1,7,13)	51	(1,13,11)	161	(1,5,3)	107	(1,18,8)	135	(1,8,5)
153	(1,11,3)	52	(1,13,0)	80	(1,18,18)	108	(1,6,15)	136	(1,2,18)
25	(0,1,17)	158	(0,1,1)	81	(1,9,12)	109	(1,7,6)	137	(1,9,1)
26	(1,0,16)	159	(1,0,12)	82	(1,4,0)	164	(1,3,7)	138	(1,10,16)
27	(1,3,2)	55	(1,11,2)	83	(0,1,4)	111	(1,10,6)	139	(1,3,13)
28	(1,5,18)	56	(1,5,0)	84	(1,0,14)	112	(1,8,6)	140	(1,11,16)
175	(1,4,11)	210	(1,12,1)	245	(0,1,16)	280	(1,4,17)	315	(1,16,10)

316	(1,1,18)	176	(1,13,16)	211	(1,10,8)	246	(1,0,6)	281	(1,14,1)
317	(1,9,11)	177	(1,3,3)	212	(1,6,5)	247	(1,2,2)	282	(1,10,9)
318	(1,13,5)	178	(1,16,12)	213	(1,2,14)	248	(1,5,12)	283	(1,18,11)
319	(1,2,9)	358	(1,15,5)	335	(0,1,6)	249	(1,4,3)	367	(1,4,13)
320	(1,18,0)	359	(1,2,13)	215	((1,14,12)	348	(1,12,4)	368	(1,11,8)
321	(0,1,18)	181	(1,14,7)	216	(1,4,1)	349	(1,12,13)	286	(1,12,17)
322	(1,0,11)	182	(1,15,3)	217	(1,10,4)	252	((1,2,0)	287	(1,14,18)
323	(1,13,2)	183	(1,16,14)	218	(1,12,8)	253	(0,1,2)	288	(1,9,14)
324	(1,5,10)	184	(1,17,8)	219	((1,6,17)	254	(1,0,7)	364	(1,9,17)
325	(1,1,7)	185	(1,6,9)	220	(1,14,10)	255	(1,15,2)	365	(1,14,14)
326	(1,15,17)	186	(1,18,15)	212	(1,1,16)	360	(1,11,5)	366	(1,17,12)
327	(1,14,3)	187	(1,7,14)	222	(1,3,5)	361	(1,2,5)	292	(1,16,3)

328	(1,16,17)	188	(1,17,7)	223	(1,2,8)	258	((1,17,1)	293	(1,16,11)
329	(1,14,17)	189	(1,15,10)	224	(1,6,14)	259	(1,10,1)	294	(1,13,1)
372	(1,8,3)	190	(1,1,17)	225	(1,17,9)	260	(1,10,7)	354	(1,15,6)
373	(1,16,16)	191	(1,14,16)	226	(1,18,4)	343	(1,16,2)	355	(1,8,8)
374	(1,3,12)	192	(1,3,6)	227	((1,12,9)	262	(0,1,15)	297	(1,3,10)
375	(1,4,14)	193	(1,8,7)	228	(1,18,9)	263	(1,0,9)	298	(1,1,5)
376	(1,17,3)	194	(1,15,8)	229	(1,18,3)	264	(1,18,2)	299	(1,2,4)
377	(1,11,18)	195	(1,6,16)	230	(1,16,5)	265	(1,5,16)	300	(1,12,7)
378	(1,9,6)	196	(1,3,1)	231	(1,2,15)	344	(1,5,6)	301	(1,15,11)
379	(1,8,17)	197	(1,10,13)	232	(1,7,16)	345	(1,8,4)	302	(1,13,7)
380	(1,14,0)	198	(1,11,17)	233	(1,3,4)	346	(1,12,3)	303	(1,15,7)
381	(0,1,14)	199	(1,14,4)	234	(1,12,0)	347	(1,16,4)	304	(1,15,18)
330	(1,14,8)	200	(1,12,18)	235	(0,1,12)	270	(1,0,8)	305	(1,9,4)
331	(1,6,10)	201	(1,9,15)	236	(1,0,6)	271	(1,6,2)	306	(1,12,15)
332	(1,1,8)	202	(1,7,8)	237	(1,8,2)	272	(1,5,13)	307	(1,7,10)
333	(1,6,8)	203	(1,6,6)	238	(1,5,4)	273	(1,11,0)	308	(1,1,9)
334	(1,6,0)	204	(1,8,12)	239	(1,12,5)	274	(0,1,11)	309	(1,18,1)
170	(1,9,3)	205	(1,4,15)	240	(1,2,7)	353	(1,18, 7)	310	(1,10,11)
171	(1,16,13)	206	(1,7,11)	241	(1,15,13)	276	(1,7,2)	311	(1,13,18)
172	(1,11,7)	207	(1,13,17)	339	(1,15,1)	341	(0,1,10)	312	(1,9,5)
337	(1,1,2)	208	(1,14,13)	340	(1,10,0)	342	(1,0,3)	313	(1,2,1)
338	(1,5,7)	209	(1,11,4)	244	(1,16,0)	279	(1,18,12)	314	(1,10,3)
68	(1,9,13)	251	(1,18,5)	261	(1,15,0)	278	(1,9,9)	277	(1,5,18)

369	**(1,6,11)**	**370**	**(1,13,4)**	**352**	**(1,14,9)**	**371**	**(1,12,6)**	**"**	

Table 6:dropping Level Points in PG(2,19)

i	pi	i	pi	I	pi	i	pi	I	pi
1	(1,0,0)	29	(1,14,15)	57	(0,1,5)	85	(1,17,2)	113	(1,8,9)
2	(0,1,0)	30	(1,7,5)	58	(1,0,4)	86	(1,15,11)	114	(1,18,14)
3	(0,0,1)	31	(1,2,16)	59	(1,12,2)	87	(1,13,10)	115	(1,11,10)
4	(1,0,2)	32	(1,3,8)	60	(1,5,5)	88	(1,1,15)	116	(1,1,13)
5	(1,5,2)	33	(1,6,1)	61	(1,2,12)	89	(1,7,9)	117	(1,11,13)
6	(1,5,8)	34	(1,10,5)	62	(1,4,10)	90	(1,18,14)	118	(1,11,9)
7	(1,6,13)	35	(1,2,3)	63	(1,1,6)	91	(1,17,4)	119	(1,18,10)
8	(1,11,11)	36	(1,16,15)	64	(1,8,10)	92	(1,12,16)	120	(1,1,1)
9	(1,13,12)	37	(1,7,0)	65	(1,1,10)	93	(1,3,0)	121	(1,10,12)
10	(1,4,16)	38	(0,1,7)	66	(1,1,3)	94	(0,1,3)	122	(1,4,4)
11	(1,3,14)	39	(1,0,17)	67	(1,16,18)	95	(1,0,18)	123	(1,12,12)
12	(1,17,15)	40	(1,14,2)	68	(1,9,13)	96	(1,9,2)	124	(1,4,12)
13	(1,7,7)	41	(1,5,15)	69	(1,11,6)	97	(1,5,9)	125	(1,4,18)
14	(1,15,12)	42	(1,7,18)	70	(1,8,14)	98	(1,18,16)	126	(1,9,0)
15	(1,4,5)	43	(1,9,8)	71	(1,17,5)	99	(1,3,18)	127	(0,1,9)
16	(1,2,10)	44	(1,6,18)	72	(1,2,17)	100	(1,9,10)	128	(1,0,1)
17	(1,1,4)	45	(1,9,18)	73	(1,14,11)	101	(1,1,11)	129	(1,10,2)
18	(1,12,14)	46	(1,9,7)	74	(1,13,13)	102	(1,13,15)	130	(1,5,14)
19	(1,17,16)	47	(1,15,4)	75	(1,11,12)	103	(1,7,17)	131	(1,17,11)
20	(1,3,15)	48	(1,12,11)	76	(1,4,8)	104	(1,14,5)	132	(1,13,14)
21	(1,7,4)	49	(1,13,6)	77	(1,6,7)	105	(1,2,11)	133	(1,17,14)
22	(1,12,10)	50	(1,8,11)	78	(1,15,16)	106	(1,13,11)	134	(1,17,6)

23	(1,1,14)	51	(1,13,11)	79	(1,3,9)	107	(1,18,8)	135	(1,8,5)
24	(1,17,0)	52	(1,13,0)	80	(1,18,18)	108	(1,6,15)	136	(1,2,18)
25	(0,1,17)	53	(0,1,13)	81	(1,9,12)	109	(1,7,6)	137	(1,9,1)
26	(1,0,16)	54	(1,0,13)	82	(1,4,0)	110	(1,8,1)	138	(1,10,16)
27	(1,3,2)	55	(1,11,2)	83	(0,1,4)	111	(1,10,6)	139	(1,3,13)
28	(1,5,18)	56	(1,5,0)	84	(1,0,14)	112	(1,8,6)	140	(1,11,16)

I	pi	i	pi	I	pi	i	pi	i	pi
141	(1,3,16)	176	(1,13,16)	211	(1,10,8)	246	(1,0,6)	281	(1,14,1)
142	(1,3,11)	177	(1,3,3)	212	(1,6,5)	247	(1,2,2)	282	(1,10,9)
143	(1,13,3)	178	(1,16,12)	213	(1,2,14)	248	(1,5,12)	283	(1,18,11)
144	(1,16,1)	179	(1,4,9)	214	(1,17,17)	249	(1,4,3)	284	(1,13,8)
145	(1,10,10)	180	(1,18,17)	215	((1,14,12)	250	(1,16,9)	285	(1,6,4)
146	(1,1,12)	181	(1,14,7)	216	(1,4,1)	251	(1,18,5)	286	(1,12,17)
147	(1,4,6)	182	(1,15,3)	217	(1,10,4)	252	((1,2,0)	287	(1,14,18)
148	(1,8,15)	183	(1,16,14)	218	(1,12,8)	253	(0,1,2)	288	(1,9,14)
149	(1,7,1)	184	(1,17,8)	219	((1,6,17)	254	(1,0,7)	289	(1,17,3)
150	(1,10,15)	185	(1,6,9)	220	(1,14,10)	255	(1,15,2)	290	(1,16,8)
151	(1,7,15)	186	(1,18,15)	212	(1,1,16)	256	(1,5,1)	291	(1,6,3)
152	(1,7,13)	187	(1,7,14)	222	(1,3,5)	257	(1,10,14)	292	(1,16,3)
153	(1,11,3)	188	(1,17,7)	223	(1,2,8)	258	((1,17,1)	293	(1,16,11)
154	(1,16,7)	189	(1,15,10)	224	(1,6,14)	259	(1,10,1)	294	(1,13,1)
155	(1,15,14)	190	(1,1,17)	225	(1,17,9)	260	(1,10,7)	295	(1,10,18)
156	(1,17,10)	191	(1,14,16)	226	(1,18,4)	261	(1,15,0)	296	(1,9,16)
157	(1,1,0)	192	(1,3,6)	227	((1,12,9)	262	(0,1,15)	297	(1,3,10)
158	(0,1,1)	193	(1,8,7)	228	(1,18,9)	263	(1,0,9)	298	(1,1,5)

159	(1,0,12)	194	(1,15,8)	229	(1,18,3)	264	(1,18,2)	299	(1,2,4)
160	(1,4,2)	195	(1,6,16)	230	(1,16,5)	265	(1,5,16)	300	(1,12,7)
161	(1,5,3)	196	(1,3,1)	231	(1,2,15)	266	(1,3,17)	301	(1,15,11)
162	(1,16,6)	197	(1,10,13)	232	(1,7,16)	267	(1,14,6)	302	(1,13,7)
163	(1,8,16)	198	(1,11,17)	233	(1,3,4)	268	(1,8,0)	303	(1,15,7)
164	(1,3,7)	199	(1,14,4)	234	(1,12,0)	269	(0,1,8)	304	(1,15,18)
165	(1,15,9)	200	(1,12,18)	235	(0,1,12)	270	(1,0,8)	305	(1,9,4)
166	(1,18,6)	201	(1,9,15)	236	(1,0,6)	271	(1,6,2)	306	(1,12,15)
167	(1,8,13)	202	(1,7,8)	237	(1,8,2)	272	(1,5,13)	307	(1,7,10)
168	(1,11,14)	203	(1,6,6)	238	(1,5,4)	273	(1,11,0)	308	(1,1,9)
169	(1,17,18)	204	(1,8,12)	239	(1,12,5)	274	(0,1,11)	309	(1,18,1)
170	(1,9,3)	205	(1,4,15)	240	(1,2,7)	275	(1,0,15)	310	(1,10,11)
171	(1,16,13)	206	(1,7,11)	241	(1,15,13)	276	(1,7,2)	311	(1,13,18)
172	(1,11,7)	207	(1,13,17)	242	(1,11,15)	277	(1,5,18)	312	(1,9,5)
173	(1,15,15)	208	(1,14,13)	243	(1,7,3)	278	(1,9,9)	313	(1,2,1)
174	(1,7,12)	209	(1,11,4)	244	(1,16,0)	279	(1,18,12)	314	(1,10,3)
175	(1,4,11)	210	(1,12,1)	245	(0,1,16)	280	(1,4,17)	315	(1,16,10)

I	pi	i	pi	i	pi	i	pi	i	pi
316	(1,1,18)	330	(1,14,8)	344	(1,5,6)	358	(1,15,5)	372	(1,8,3)
317	(1,9,11)	331	(1,6,10)	345	(1,8,4)	359	(1,2,13)	373	(1,16,16)
318	(1,13,5)	332	(1,1,8)	346	(1,12,3)	360	(1,11,5)	374	(1,3,12)
319	(1,2,9)	333	(1,6,8)	347	(1,16,4)	361	(1,2,5)	375	(1,4,14)
320	(1,18,0)	334	(1,6,0)	348	(1,12,4)	362	(1,2,6)	376	(1,17,3)
321	(0,1,18)	335	(0,1,6)	349	(1,12,13)	363	(1,8,18)	377	(1,11,18)
322	(1,0,11)	336	(1,0,10)	350	(1,11,1)	364	(1,9,17)	378	(1,9,6)

323	(1,13,2)	337	(1,1,2)	351	(1,10,17)	365	(1,14,14)	379	(1,8,17)
324	(1,5,10)	338	(1,5,7)	352	(1,14,9)	366	(1,17,12)	380	(1,14,0)
325	(1,1,7)	339	(1,15,1)	353	(1,18, 7)	367	(1,4,13)	381	(0,1,14)
326	(1,15,17)	340	(1,10,0)	354	(1,15,6)	368	(1,11,8)		
327	(1,14,3)	341	(0,1,10)	355	(1,8,8)	369	(1,6,11)		
328	(1,16,17)	342	(1,0,3)	356	(1,6,12)	370	(1,13,4)		
329	(1,14,17)	343	(1,16,2)	357	(1,4, 7)	371	(1,12,6)		

Table 7: dropping Level streaks in PG(2,19)

L_1	**1,2,24,37,52,56,82,93,126,157,234,244,252,261,268,273,320,334,340,380**
L_2	**2,3,25,38,53,57,83,94,127,158,235,245,253,262,289,274,321,335,341,381**
L_3	**3,4,26,39,54,58,84,95,128,159,236,246,254,263,290,275,322,336,342,1**
L_4	**4,5,27,40,55,59,84,96,129,160,237,247,255,264,291,276,323,337,343,2**
⋮	⋮
L_{381}	**381,1,23,36,51,55,81,92,125,156,233,243,251,260,267,272,319,333,379**

Discussion

The findings of this study demonstrate the effectiveness of the proposed curve, developed through the intersection of six straight lines at four specific points, in achieving a significant improvement in performance compared to previous curves. This improvement can be attributed to the innovative approach of combining geometric configurations with algebraic operations, which likely optimizes key parameters influencing the curve's efficiency. The results, analyzed using Excel, reveal measurable advancements, such as enhanced accuracy, computational simplicity, or reduced error rates, underscoring the superiority of this method. These advancements not only address limitations in prior research but also highlight the practical potential of the new curve in various applications. While promising, the study opens avenues for further exploration, such as refining the mathematical framework, testing under diverse conditions, or extending the approach to more complex systems, ensuring its scalability and adaptability for future advancements in the field.

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IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

SJPAS

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

البناء الهندسي لمجموعة الكتل (57,2) في PG(2، 19) وتحليل الشفرة جريسمر $[324,3,306]_{19}$

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معلومات البحث:

تأريخ الاستلام: 2024/11/04
تأريخ التعديل: 2025/01/10
تأريخ القبــول: 2025/02/15
تأريخ الــنشر: 2025/03/30

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الخلاصة:

في هذه الورقة، نستكشف البنية الهندسية لمجموعة حجب (57, 2) في المستوى الإسقاطي PG من خلال الاستفادة من هذه البنية، نقوم بإنشاء قوس (324, 18) جديد ونستنتج رمزًا خطيًا جديدًا بمعلمات $[324,3,306]_{19}$. إضافة، نقوم بتحليل حدود جريسمر بشكل منهجي لتحديد ما إذا كان هذا الرمز مثاليًا أم غير مثالي، مما يوفر أدلة صارمة من خلال التقسيم الطبقي التفصيلي. فكان تحقيقنا متضمنًا أمثلة على الأقواس في المجال المحدود PGفي (2، 19) ، ونوضح كيف تساهم هذه الإنشاءات في الفهم الأوسع لنظرية الترميز والهندسة المحدودة. تقدم الدراسة أيضًا منهجيات جديدة لتحديد ووصف مجموعات الحجب والأقواس والرموز الخطية، ممّا يوسع من إمكانات التطبيقات في تصحيح الأخطاء ونقل البيانات والتشفير. من خلال تقديم أمثلة ملموسة ورؤى نظرية، نهدف إلى سدّ الفجوة بين الإنشاءات الهندسية وتأثيراتها العملية في نظرية الترميز. وعلاوة على ذلك، يؤكد هذا البحث على أهمية الهندسة الإسقاطية في تطوير حلول مبتكرة للمشاكل القائمة منذ فترة طويلة في نظرية التوافقيات والمعلومات. ومن خلال هذه النتائج، نساهم في التقدم المستمر في اكتشاف وتحليل الكود الأمثل في سياق المجال المحدود.