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The Novel Odd Generalized Exponential-G Family: Statistical Properties and Applications

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Abstract

This article presents The Novel Odd Generalized Exponential-G family (NOGE-G), a recently discovered distribution family. The statistical properties of a new family, which includes the following, have been studied. survival function and hazard function. We considered the cumulative hazard function, its moments, the characteristic function, the quantile function, and renyi entropy. Also considered was the estimate from the Maximum Likelihood method for parameter estimation. We expanded and combined the inverse Weibull (IW) distribution with it to explore the asymptotic behaviour of the estimates under the estimating method. This resulted in a new extended distribution, which we then simulated using Monte Carlo simulations to estimate the unknown parameters The OGEIW distribution is shown to be genuinely practical and beneficial by its application to two actual data sets.

Introduction:

The main goal of statistics is to discover reliable probability distributions that can serve as models for real-world events. The use of probability distributions allows for the modeling of naturally occurring events that exhibit uncertainty and risk. Nevertheless, numerous probability distributions have been developed due to the complexity and difficulty of representing natural events using standard distributions. Regardless, recognized probability distributions still fail to adequately capture some data from natural events; hence, generalized probability distributions have been expanded and modified to better fit these data. Alizadeh, et al. introduced the extended odd Weibull-G family: properties and applications [1]. Cordeiro, et al. introduced the odd Lomax generator of distributions: properties, estimation, and applications [2]. Afify, et al. introduced The odd Dagum family of distributions: Properties and applications [3]. Al-Moisheer, et al. studied odd inverse power generalized Weibull generated family of distributions: Properties and applications. [4]. Rasekhi, et al. studied the odd log-logistic Weibull-G family of distributions with regression and financial risk models [5]. Jamal, et al. studied the generalized odd linear exponential family of distributions with applications to reliability theory [6]. Suleiman, et al. studied the odd beta prime-G family of probability distributions: properties and applications to engineering and environmental data [7]. Suleiman, et al. studied A novel odd beta prime-logistic distribution: desirable mathematical properties and applications to engineering and environmental data [8]. Numerous studies have used various estimating approaches, as shown by references [9], [10], [11], and [12].

The transformed-transformer family (T-X) was proposed by Alzaatrah et al., who used a novel approach to generate a generic mode, Suppose that $W(G(X,\varsigma)$ is the cumulative distribution function(cdf) of the random variable X using a vector of parameters ς that satisfies the following conditions:

- $W(G(X,\varsigma)$ is bound between c and d
- $W(G(X,\varsigma)$ is differentiable and increasingly monotonous
- $W(G(X,\varsigma) \to \text{c}$ as $x \longrightarrow -\infty$ and $W(G(X,\varsigma) \to d$ as $x \longrightarrow \infty$.

The Family Proposal is based on the T-X generator technique described in Alzaatrah et al [13], with a cumulative distribution function (cdf) as follows:

$F(x) = \int_{0}^{W(G(X,\theta)} r(t)dt \quad t \in \square$, where the pdf of a random variable T is denoted by r(t) and the function of the cdf for the baseline function of a random variable X is represented by. With the goal of creating the NOGE-G family using the T-X family method, let $r(t) = \theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\theta-1}$ and be the baseline cdf, $G(X,\theta)$ in this case

This study introduces a new family named NOGE-G. The purpose of the study is to derive certain mathematical and statistical properties of this novel family. Our goal is to construct a new family by using the Alzaatrah et al. approach to define numerous new distributions for coping with large tail data. The NOGE-G family's overarching goal is for distribution generation to be more flexible for real-world data. The study's motivation stemmed from the inability to model data for certain real-world phenomena, which served as a catalyst to address this issue. Scientific contribution Create a new family that encompasses a variety of distributions, each capable of playing a more flexible and effective role in modeling real-world data. The research outline includes an introduction in Section 1. Section 2 presents a proposed family of NOGE-G distributions. Section 3 includes new family extensions. Section 4 introduces and discusses important statistical and mathematical features of the proposed family. Section 5 includes parameter estimation, and Section 6 includes the proposed distribution.

2. Family Proposal

The Generalized Exponential distribution has the CDF and PDF [14]

$$F(x,\theta,\lambda) = \left(1-e^{-\lambda x}\right)^{\theta} \qquad x \geq 0, \theta, \lambda > 0 \qquad (1)$$

$$f(x,\theta,\lambda) = \theta\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\theta-1} \qquad (2)$$

The Family Proposal is based on the T-X generator technique described in Alzaatrah et al [13], with a

The cdf and pdf of the NOGE-G distribution family can be expressed as a result of these findings.

$$G(x,\theta,\lambda)=\left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta} \quad , \ x \geq 0, \theta, \lambda > 0 \qquad (3)$$

$$g(x,\theta,\lambda) = \theta\lambda F(x)f(x)(2-F(x))(1-F(x))^{-2}\left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta-1} e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)} \qquad (4)$$

2.1 Survival Function

How the NOGE-G family Survival Function. The following formula can be used to accomplish this:

$$S(x) = 1 - G(x,\theta,\lambda)$$

where $G(\mathrm{x},\theta,\lambda)$ was defined in equation (3) we obtain

$$S(x,\theta,\lambda) = 1 - \left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta} , x \geq 0, \theta, \lambda > 0 \qquad (5)$$

2.2 Hazard Function

The NOGE-G family's hazard function is

$$h(x) = \frac{g(x,\theta,\lambda)}{\mathrm{S}(x,\theta,\lambda)}$$

where $g(x,\theta,\lambda)$ and $\mathrm{S}(x)$ was defined in equation (4) and (5) we obtain

$$h(x,\theta,\lambda)=\frac{\theta\lambda F(x)\,f(x)\big(2-F(x)\big)\big(1-F(x)\big)^{-2}\left[1-e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}\right]^{\theta-1}e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}}{1-\left[1-e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}\right]^{\theta}} \qquad (6)$$

2.3 Cumulative hazard function

Cumulative hazard function of NOGE-G family you may calculate it using this formula.

$H(x)=-\ln(1-G(x))$

where $G(x,\theta,\lambda)$ was defined in equation (3) we obtain

$$H(x,\theta,\lambda)=-\ln\left(1-\left[1-e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}\right]^{\theta}\right) \qquad (7)$$

3. Expansion CDF and PDF of NOGE-G

In this section, extend the pdf and cdf of the NOGE-G family derivative, which is useful for the study of many statistical Characteristics of the NOGE-G family. The generalized binomial is used to find a CDF, and PDF, expression

3.1 Expansion CDF of NOGE-G

To expand the CDF for NOGE-G by using Eq (3) , by Invoking the generalized binomial model: [14]

$$\left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta}=\sum_{r=0}^{\infty}(-1)^r\binom{\theta}{r}e^{-\lambda r\left(\frac{(F(\mathrm{x})^2)}{1-F(x)}\right)}$$

for the use of the generalized binomial theorem

$$e^{-\lambda r\left(\frac{(F(x))^2}{1-F(x)}\right)}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\left(\lambda r\left(\frac{(F(x))^2}{1-F(x)}\right)\right)^k$$

$$G(x,\lambda,\theta)=\sum_{r=k=0}^{\infty}\frac{(-1)^{r+k}}{k!}\binom{\theta}{r}\left(\lambda r\left(\frac{(F(x))^{2}}{1-F(x)}\right)\right)^{k}$$

$$G(x,\lambda,\theta)=\sum_{r=k=0}^{\infty}\frac{(-1)^{r+k}\lambda^{k}r^{k}}{k!}\binom{\theta}{r}\left(F(x)^{2k}\right)(1-F(x))^{-k}$$

By Invoking the generalized binomial model

$$(1-F(x))^{-k}=\sum_{m=0}^{\infty}\frac{\Gamma(k+m)}{m!\Gamma(k)}F(x)^{m}$$

The extender cdf of NOGE-G is obtained as follows

$$G(x,\lambda,\theta)=\sum_{r=k=m=0}^{\infty}\frac{(-1)^{r+k}\lambda^{k}r^{k}\Gamma(k+m)}{k!\mathrm{m}!\Gamma(\mathrm{k})}\binom{\theta}{r}(F(x))^{2k+m} \tag{8}$$

3.2 Expansion (Pdf) of NOGE-G

Now to expand the (Pdf) for NOGE-G by using Eq.(4) as.

Now by Invoking the generalized binomial model

$$\left[1-e^{-\lambda\left(\frac{(F(x))^{2}}{1-F(x)}\right)}\right]^{\theta-1}=\sum_{r=0}^{\infty}(-1)^{r}\binom{\theta-1}{r}e^{-\lambda r\left(\frac{(F(x))^{2}}{1-F(x)}\right)}$$

$$g(x,\theta,\lambda)=\sum_{r=0}^{\infty}(-1)^{r}\binom{\theta-1}{r}\theta\lambda F(\mathrm{x})\mathrm{f}(\mathrm{x})(2-F(x))(1-F(x))^{-2}e^{-\lambda r\left(\frac{(F(x))^{2}}{1-F(x)}\right)}e^{-\lambda\left(\frac{(F(\mathrm{x}))^{2}}{1-F(x)}\right)}$$

$$=\sum_{r=0}^{\infty}(-1)^{r}\binom{\theta-1}{r}\theta\lambda F(x)f(x)(2-F(x))(1-F(x))^{-2}e^{-\lambda(1+r)\left(\frac{(F(x))^{2}}{1-F(x)}\right)}$$

Now by Invoking the generalized binomial theorem

$$e^{-\lambda(1+r)\left(\frac{(F(x))^{2}}{1-F(x)}\right)}=\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\lambda^{k}(1+r)^{k}(F(x))^{2k}(1-F(x))^{-k}$$

$$g(x,\theta,\lambda)=\sum_{r=k=0}^{\infty}\frac{(-1)^{r+k}}{k!}\theta\lambda^{k+1}(1+r)^{k}\,\mathrm{f}(\mathrm{x})(1-F(x))^{-k-2}(2-F(x))(F(x))^{2k+1}$$

Now by Invoking the generalized binomial theorem

$$\left(1-F(x)\right)^{-(k+2)}=\sum_{m=0}^{\infty}\frac{\Gamma(k+2+m)}{m!\Gamma(k+2)}F^{m}(x)$$

The extender pdf of OGE-G is obtained as follows

$$g(x,\lambda,\theta)=\Upsilon\left(2f(x)\left(F(x)\right)^{2k+m+1}-f(x)\left(F(x)\right)^{2k+m+2}\right) \qquad (9)$$

Where $\Upsilon=\sum_{r=k=m=0}^{\infty}\frac{\theta(-1)^{r+k}\binom{\theta-1}{r}\lambda^{k+1}\Gamma(k+2+m)}{r!m!\Gamma(k+2)}$

4 Mathematical properties of NOGE-G

Here we derive a few structural features of the NOGE-G family, such as the Moments, Moment Generating Function, Characteristic function, The Quantile function.

4.1 Moments

Theorem: If X □ $NOGE-G$ family then r^{th} moment of X is given by:

$$M_r=\psi_{r,k,m}\int_0^{\infty}2X^r f(x)\left(F(x)\right)^{2k+m+1}dx-\psi_{r,k,m}\int_0^{\infty}X^{r}f(x)\left(F(x)\right)^{2k+m+2}dx$$

Proof: At the r^{th} moment when a random variable X is described as

$$M_r=E(X^r)=\int_0^{\infty}X^r g(x,\theta,\lambda)\,\mathrm{dx}$$

$g(x,\theta,\lambda)$ Provided in equation (9) for

$$M_r=\psi_{r,k,m}\int_0^{\infty}2X^r f(x)\left(F(x)\right)^{2k+m+1}dx-\psi_{r,k,m}\int_0^{\infty}X^{r}f(x)\left(F(x)\right)^{2k+m+2}dx \quad (10)$$

By using equation (10), we can determine the $\dot{\mu}_1, \dot{\mu}_2, \dot{\mu}_3, \dot{\mu}_4$, variance $V=E(X^2)-\left(E(X)\right)^2$, skewness $=\frac{\mu_3}{\mu_2^{\left(\frac{3}{2}\right)}}$, and kurtosis $=\frac{\mu_4}{\mu_2^2}$ of the NOGE-G family.

4.2 Moment Generating Function

Theorem: If $X\sim NOGE-G$ family then MGF of X is given by

$$M_X(t)=\Upsilon\int_{-\infty}^{\infty}2e^{tx}f(x)\left(F(x)\right)^{2k+m+1}dx-\Upsilon\int_{-\infty}^{\infty}e^{tx}f(x)\left(F(x)\right)^{2k+m+2}dx$$

Proof: The moment-generating functions (MGF) of the NOGE-G family are as follows:

$$M_X(t)=E(e^{tx})=\int_{-\infty}^{\infty}e^{tx}g(x,\theta,\lambda)\,\mathrm{dx}$$

$g(x,\theta,\lambda)$ Provided in equation (9) for

$$M_X(t)=\Upsilon\int_{-\infty}^{\infty} 2e^{tx} f(x)\left(F(x)\right)^{2k+m+1}dx-\Upsilon\int_{-\infty}^{\infty} e^{tx} f(x)\left(F(x)\right)^{2k+m+2}dx \qquad (11)$$

4.3 Characteristic function

By using this information, we can derive the characteristic function, which is

$$Q_r(t)=E(e^{itx})=\int_0^{\infty} e^{itx} g(x,\theta,\lambda)\,\mathrm{dx}$$

where $g(x,\theta,\lambda)$ was defined in equation (9) we obtain

$$Q_r(t)=\Upsilon\int_0^{\infty} 2e^{itx} f(x)\left(F(x)\right)^{2k+m+1}dx-\Upsilon\int_0^{\infty} e^{itx} f(x)\left(F(x)\right)^{2k+m+2}dx \qquad (12)$$

4.4 The Quantile function

The following is the inverted form of the supplied in (3) function that yields the quantile function for the NOGE-G distribution family: [15]

$$u=\left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta}$$

Then the quantile for the NOGR-G family is:

$$F(x)=\frac{-\frac{-1}{\lambda}\ln\left(1-u^{\frac{1}{\theta}}\right)\mp\sqrt{\left(\frac{-1}{\lambda}\ln\left(1-u^{\frac{1}{\theta}}\right)\right)^2-\frac{4}{\lambda}\ln\left(1-u^{\frac{1}{\theta}}\right)}}{2} \qquad (13)$$

4.5. Renyi Entropy

If $X \sim NOGE-G$ family then Renyi Entropy of X is given by

$$\boldsymbol{I_R(\omega)} = \frac{\mathbf{1}}{\mathbf{1-\omega}} log\int_{\mathbf{0}}^{\infty} \boldsymbol{g(x,\theta,\lambda)^{\omega}}\,dx \quad ,\omega>0,\omega\neq 0$$

Substituting equation (4) into the equation above:

$$\boldsymbol{I_R(\omega)} = \frac{\mathbf{1}}{\mathbf{1-\omega}}\boldsymbol{log}\int_{\mathbf{0}}^{\infty}\left(\theta\lambda F(x)f(x)(2-F(x))(1-F(x))^{-2}\left[1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\theta-1} e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right)^{\boldsymbol{\omega}}$$

5. Parameters Estimation

To estimate parameters, the maximum likelihood method is applied to estimate the NOGE-G family's unknown parameters. Think of the NOGE-G family as a random sample with values $x_1, x_2, \ldots, x_n$ Afterwards, the probability function (Eq4) is the form taken by for (θ,λ) :[17]

$$L(\theta,\lambda\backslash x)=\prod_{i=1}^{n} g(x_i,\theta,\lambda)$$

$$=\prod_{i=1}^{n}\left[\theta\lambda F(x)\,f(x)(2-F(x))\left(1-F(x)\right)^{-2}\left(1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right)^{\theta-1} e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]$$

$$\ln L(\lambda,\theta\backslash x)=n\ln\theta+n\ln\lambda+\ln\sum_{i=1}^{n}F(x)+\ln\sum_{i=1}^{n}f(x)+\ln\sum_{i=1}^{n}(2-F(x))-2\ln\sum_{i=1}^{n}\left(1-F(x)\right)$$

$$+\left(\theta-1\right)\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right)-\lambda\sum_{i=1}^{n}\left(\frac{(F(x))^2}{1-F(x)}\right)$$

$$\frac{\partial\ln L(\lambda,\theta\backslash x)}{\partial\lambda}=\frac{n}{\lambda}+\left(\theta-1\right)\sum_{i=1}^{n}\frac{\frac{F^2(x)}{1-F(x)}e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}}{1-e^{-\lambda\left(\frac{F^2(x)}{1-F(x)}\right)}}-\sum_{i=1}^{n}\frac{F^2(x)}{1-F(x)} \tag{14}$$

$$\frac{\partial\ln L(\lambda,\theta\backslash x)}{\partial\theta}=\frac{n}{\theta}+\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\left(\frac{(F(x))^2}{1-F(x)}\right)}\right) \tag{15}$$

We can determine the parameter values that maximize the likelihood by setting the score function to zero for the equations $\frac{\partial}{\partial\theta}$ and $\frac{\partial}{\partial\lambda}$. But it's not always easy to solve these equations algebraically. For this reason, we use numerically iterative approaches, like algorithms of the Newton-Raphson type, which are implemented in the R package "Newdistns." As will be discussed in the section that follows, this method enables us to derive parameter estimators.

6. Proposed distribution

In this part, an expansion proposal is introduced for the inverse Weibull (IW) Distribution with shape parameters (q) and (d). Then cdf and pdf of the IW distribution are defined as follows:[18]

$$F(x)_{IW}=e^{-dx^{-q}} \tag{16}$$

And

$$f(x)_{IW} = dqx^{-q-1}e^{-dx^{-q}} \quad , x, d, q > 0 \qquad (17)$$

which is substituted in equation 3 and 4, the new distribution, called NOGEIW distribution, which has the cdf and pdf

$$G(x)_{NOGEIW} = \left(1 - e^{-\lambda\left(\frac{\left(e^{-dx^{-q}}\right)^{2}}{1-e^{-dx^{-q}}}\right)}\right)^{\theta} \quad , x, {}^{\theta}, d, q > 0 \qquad (18)$$

$$g(x)_{NOGEIW} = \theta\lambda dqx^{-q-1}e^{-2dx^{-q}}\left(2 - e^{-dx^{-q}}\right)\left(1 - e^{-dx^{-q}}\right)^{-2}\left(1 - e^{-\lambda\left(\frac{\left(e^{-dx^{-q}}\right)^{2}}{1-e^{-dx^{-q}}}\right)}\right)^{\theta-1} e^{-\lambda\left(\frac{\left(e^{-dx^{-q}}\right)^{2}}{1-e^{-dx^{-q}}}\right)} (27)$$

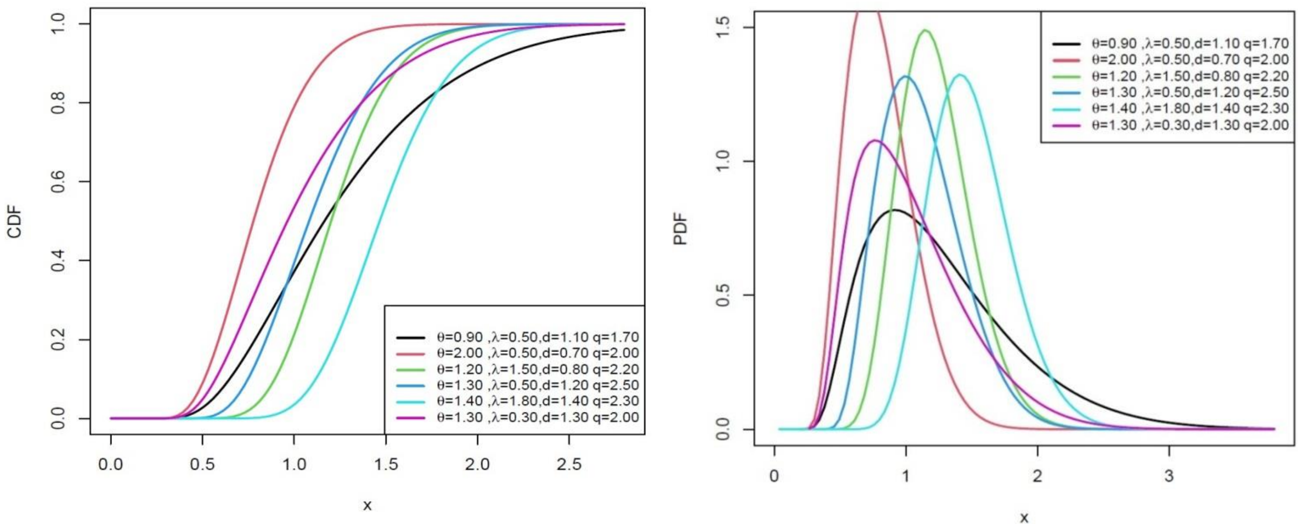

Fig 1: *the cdf and pdf for the OGEIW distribution*

7. Simulation

To analyze the asymptotic behavior of MLEs for NOGEIW Distribution parameters in this part by means of a Monte Carlo experiment. In this study, four distinct parameter combinations are examined: (θ =0.4, λ =0.5, d =0.7, q =0.7), (θ =0.3, λ =0.3, d =0.6, q =0.4), (θ =0.4, λ =0.5, d =0.7, q =0.6), and (θ =4, λ =0.4, d =0.8, q =0.9). A thousand times through the process, we look at four distinct sample sizes: 10, 20, 80, and 100. In general, there are several different scales used in simulations. In this study, two comparison scales were used, namely Root Mean Square Error (MRSE) and bias, where, $Bias(\hat{\theta})=E(\hat{\theta})-\theta$,where the estimated value of the parameter θ .

Table 1. Monte Carlo Simulation Results for the NOGEIW Distribution.

		(θ =0.4, λ =0.5, d =0.7, q =0.7)			(θ =0.3, λ =0.3, d =0.6, q =0.4)		
parameter	Sample Size	Mean	RMSE	Abias	Mean	RMSE	Abias
θ	100	1.0923	1.8884	0.6923	0.5048	1.6714	0.2048
	200	0.6509	1.4936	0.2509	0.3342	0.2814	0.0342
	300	0.5206	0.7231	0.1206	0.3239	0.2462	0.0239
	500	0.4668	0.3680	0.0668	0.3205	0.2321	0.0205
λ	100	0.6277	0.8199	0.1277	0.4417	0.7498	0.1417
	200	0.5691	0.5689	0.0691	0.3178	0.1054	0.0178
	300	0.5283	0.4170	0.0283	0.3085	0.0956	0.0085
	500	0.0516	0.3546	0.0168	0.3084	0.0605	0.0084
d	100	0.8040	0.5201	0.1040	0.6574	0.3361	0.0425
	200	0.7987	0.4783	0.0987	0.5855	0.2991	0.0144
	300	0.7885	0.4390	0.0885	0.5846	0.2955	0.0053
	500	0.7531	0.3656	0.0531	0.5792	0.2404	0.0027
q	100	0.7736	0.2550	0.0736	0.4205	0.1028	0.0205
	200	0.7323	0.1776	0.0323	0.4123	0.0719	0.0123
	300	0.7290	0.1490	0.0290	0.4115	0.0592	0.0115
	500	0.7172	0.1164	0.0172	0.4074	0.0514	0.0074

Table 2. Monte Carlo Simulation Results for the NOGEIW Distribution.

		(θ =0.4, ג =0.5, d =0.7, q =0.6)			(θ =4, ג =0.4, d =0.8, q =0.9)		
parameter	Sample Size	Mean	RMSE	Abias	Mean	RMSE	Abias
θ	100	1.0663	2.2267	0.6663	0.8209	1.3720	0.4209
	200	0.6244	1.3257	0.2440	0.5148	0.8335	0.1148
	300	0.5517	0.9112	0.1517	0.4572	0.3815	0.0572
	500	0.4796	0.4266	0.0796	0.4451	0.3699	0.0451
ג	100	0.6273	0.8281	0.1273	0.5109	0.7064	0.1109
	200	0.6002	0.5805	0.1002	0.4819	0.5689	0.0819
	300	0.5511	0.4648	0.0511	0.4183	0.2666	0.0183
	500	0.5178	0.3329	0.0178	0.4070	0.1793	0.0070
d	100	0.8149	0.5250	0.1149	0.8414	0.4979	0.0414
	200	0.7670	0.4324	0.0670	0.8363	0.4551	0.0363
	300	0.7662	0.4319	0.0826	0.8270	0.4256	0.0270
	500	0.7644	0.3646	0.0644	0.8266	0.3737	0.0266
q	100	0.6517	0.2161	0.0517	0.9864	0.2949	0.0864
	200	0.6163	0.1493	0.0163	0.9511	0.2191	0.0511
	300	0.6152	0.1258	0.0152	0.9384	0.1725	0.0384
	500	0.6104	0.0978	0.0104	0.9252	0.1350	0.0252

The results presented in Tables 1 and 2 demonstrate that as the sample size (n) increases for all parameter values, the Root Mean Square Error (RMSE) decreases and approaches zero. This indicates that the estimation of the parameters was unbiased, demonstrating the effectiveness of the MLE method for the Distribution parameters.

8. Application

To demonstrate the adaptability of the new distribution, this section presents an example of using the NOGEIW distribution on a real dataset. The R program was used to estimate the model parameters. By comparing the new distribution to actual data, one can see whether it is better and more appropriate than the previous distributions using estimated model values for the following metrics: -L (log-likelihood), AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), and HQIC (Hannan-Quinn Information Criterion, uses the new distribution with actual data to prove that the suggested distribution is better and more appropriate than the next distribution.

Gompertz Inverse Weibull (GOIW) distribution [19], Truncated Exponentiated Exponential Inverse Weibull (TEEIW) distribution [20], Beta Inverse Weibull (BeIW) distribution [21], Kumaraswamy Inverse Weibull (KuIW) distribution [22], Exponential Generalized Inverse Weibull (EGIW) distribution [23], odd Generalized Rayleigh Inverse Weibull (OGRIW) distribution [24], and Inverse Weibull (IW) distribution [18].

The first dataset: The dataset comprises 30 observations of March precipitation in Minneapolis/St. Paul, recorded in inches. [25] The following values were observed.

0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05

Table 3: The values of the -2 *l*, AIC, CAIC, BIC, HQIC

Distribution	perimeter	-LL	AIC	CAIC	BIC	HQIC
NOGEIW	$\hat{\theta} = 0.1287$ $\hat{\lambda}=1.1435$ $\hat{d} = 0.2263$ $\hat{q} = 1.3779$	37.84	83.69	85.29	89.29	85.48
GOIW	$\hat{\theta} = 1.9576$ $\hat{\lambda}=2.1618$ $\hat{d} = 1.9973$ $\hat{q} = 0.8094$	37.86	83.72	85.32	89.33	85.52
TEEIW	$\hat{\theta} = 32.116$ $\hat{\lambda}=0.2508$ $\hat{d} = 8.4045$ $\hat{q} = 0.8174$	37.90	83.81	85.41	89.42	85.61
BeIW	$\hat{\theta} = 0.3386$ $\hat{\lambda}=19.835$ $\hat{d} = 6.9726$ $\hat{q} = 0.7584$	37.98	83.96	85.56	89.57	85.76
KuIW	$\hat{\theta} = 1.6524$ $\hat{\lambda}=4.7616$ $\hat{d} = 1.5723$ $\hat{q} = 0.8375$	37.15	86.32	87.92	91.93	88.12
EGIW	$\hat{\theta} = 21.999$ $\hat{\lambda}=0.3000$ $\hat{d} = 7.1975$ $\hat{q} = 0.1838$	37.95	83.90	85.50	89.50	85.69
OGRIW	$\hat{\theta} = 0.4186$ $\hat{\lambda}=-0.2668$ $\hat{d} = 1.1466$ $\hat{q} = 1.0311$	38.02	84.05	85.65	89.65	85.84
IW	$\hat{\theta} = 1.0252$ $\hat{\lambda}=1.5496$	41.91	87.83	88.27	90.63	88.73

Table 4: The values of the W, A, K-S and p-value of the data set

Distribution	NOGEIW	GOIW	TEEIW	BeIW	KuIW	EGIW	OGRIW	IW
W	0.0151	0.0153	0.0166	0.0176	0.0514	0.0178	0.0192	0.1260
A	0.1067	0.1085	0.1142	0.1197	0.3225	0.1211	0.1415	0.7721
KS	0.0708	0.0715	0.0776	0.0780	0.1076	0.0806	0.0803	0.1523
p-value	0.9982	0.9979	0.9936	0.9931	0.8774	0.9898	0.9902	0.4892

Tables 3 and 4 show that the. distribution is superior; the newly expanded and extended distribution gives a true picture due to its lowest scores on the informational and statistical standards 2 L, AIC, CAIC, BIC, HQIC, K-S, A, and W, as well as its highest p-value.

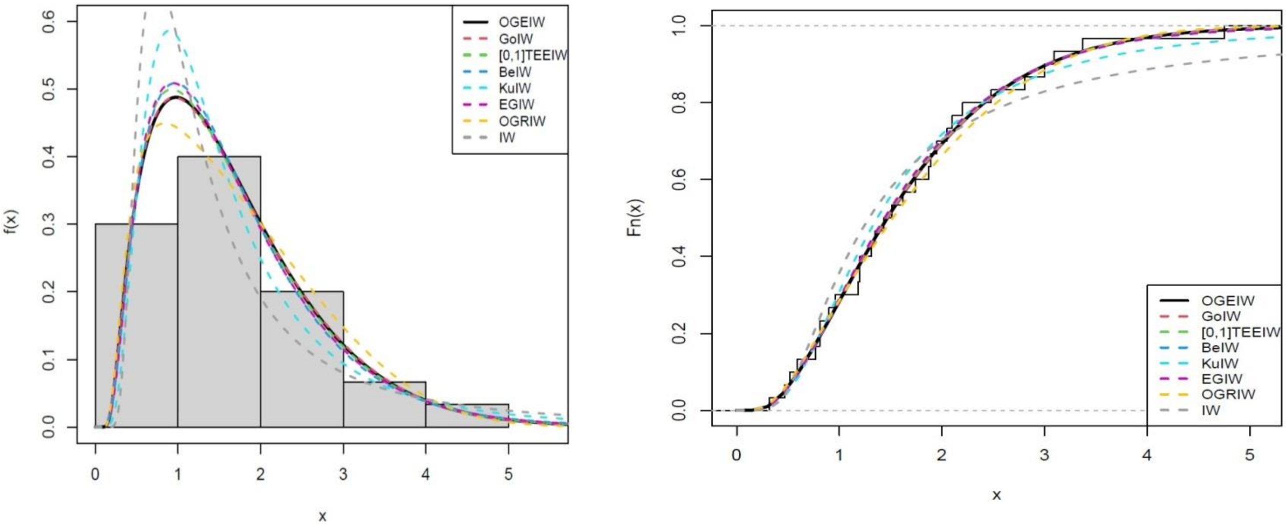

Fig 2: *the pdf and cdf for the datasets1*.

The second dataset: Variable 5 of the monthly metrics on claims for unemployment insurance is represented in the data given by the Department of Labour, Licensing, and Regulation, State of Maryland, USA, as mentioned in reference [26]. The data points are listed below:

1.29, 1.03, 1.29,1.25, 1.03, 1.11, 1.49,1.15, 1.31, 1.06,1.02, 1.38, 1.41, 1.40, 1.55, 1.49, 1.06,1.32, 1.37, 1.18, 1.36, 1.57, 1.24, 1.77,1.70, 2.03,1.84, 1.73, 1.53, 1.53, 1.66,1.45, 1.45, 1.48, 1.44, 1.64, 1.66, 1.78,1.71, 1.79, 1.66, 1.27, 2.07, 1.68, 1.92,1.82,1.93, 1.91, 1.95, 1.94, 1.56, 2.67,1.80, 1.45, 2.07, 1.59, 1.49, 1.72.

Table 5: The values of the -2 l, AIC, CAIC, BIC, HQIC

Distribution	perimeter	-LL	AIC	CAIC	BIC	HQIC
NOGEIW	$\hat{\theta} = 1.0922$ $\hat{\lambda}$=1.9671 $\hat{d} = 1.1801$ $\hat{q} = 2.2689$	13.98	35.96	36.71	44.20	39.17
GOIW	$\hat{\theta} = 1.8495$ $\hat{\lambda}$=0.8630 $\hat{d} = 4.7052$ $\hat{q} = 3.1083$	13.99	35.99	36.74	44.23	39.20
BeIW	$\hat{\theta} = 1.9821$ $\hat{\lambda}$=5.8435 $\hat{d} = 3.6367$ $\hat{q} = 2.2320$	14.58	37.19	37.94	45.43	40.40
KuIW	$\hat{\theta} = 2.4426$ $\hat{\lambda}$=5.8582 $\hat{d} = 2.4565$ $\hat{q} = 2.4380$	14.29	36.60	37.35	44.84	39.81
EGIW	$\hat{\theta} = 6.8162$ $\hat{\lambda}$=0.8778 $\hat{d} = 6.5579$ $\hat{q} = 2.3488$	14.12	36.28	37.04	44.52	39.49
OGRIW	$\hat{\theta} = 2.8843$ $\hat{\lambda}$=5.3573 $\hat{d} = 0.7058$ $\hat{q} = 0.6993$	14.04	36.09	36.84	44.33	39.30
IW	$\hat{\theta} = 4.9991$ $\hat{\lambda}$=5.0690	18.18	40.36	40.57	44.48	41.96

Table 6: The values of the W, A, K-S, and p-value of the data set

Distribution	NOGEIW	GOIW	BeIW	KuIW	EGIW	OGRIW	IW
W	0.0234	0.0313	0.0593	0.0481	0.0404	0.0242	0.1800
A	0.2454	0.3133	0.4711	0.3978	0.3485	0.2465	1.2338
KS	0.0567	0.0583	0.0776	0.0721	0.0642	0.0603	0.1102
p-value	0.9922	0.9891	0.8750	0.9236	0.9705	0.9840	0.4813

Tables 5 and 6 show that the. distribution is superior; the newly expanded and extended distribution gives a true picture due to its lowest scores on the informational and statistical standards 2 L, AIC, CAIC, BIC, HQIC, K-S, A, and W, as well as its highest p-value.

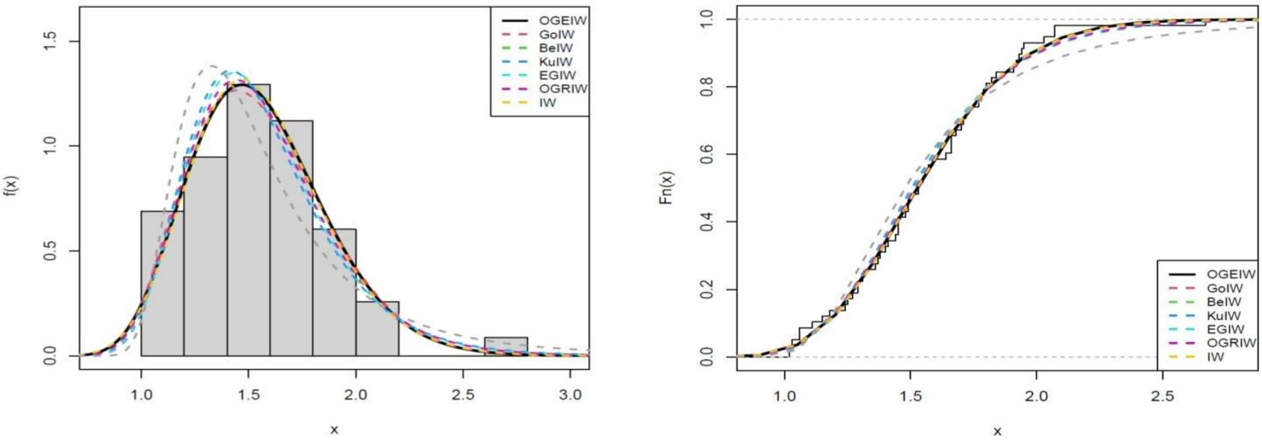

Fig 3: *the pdf and cdf for the datasets 2.*

9. CONCLUSION

A generalization of distribution Exponential, the Noval Odd Generalized Exponential-G family of distributions, has been extended. Obtained the statistical properties of the new distribution family. We also presented a special case, Odd Generalized Exponential Inverse Weibull (NOGEIW), where we employed the maximum likelihood method to discover unknown parameters and conducted Monte Carlo simulation research to examine the stability of parameter maximum likelihood estimators (MLEs), paying particular attention to average biases and root mean square errors. In comparison to the GoIW, BeIW, KuIW, EGIW, OGRIW, and IW distributions, the NOGRIW distribution shows a greater amount of tail heaviness. The results of using an NOGEIW distribution on a real-world dataset demonstrate that it is more appropriate for global data.

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العائلة الاسية المعممة الفردية الجديدة: الخصائص الاحصائية والتطبيقات

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الخلاصة:

تعرض هذه المقالة عائلة الاسية المعممة الفردية الجديدة (NOGRIW)، وهي عائلة توزيع تم اكتشافها مؤخرًا. تمت دراسة الخصائص الإحصائية لعائلة جديدة والتي تشمل ما يلي: وظيفة البقاء ووظيفة الخطر. لقد أخذنا بعين الاعتبار دالة الخطر التراكمية، لحظاتها، الدالة المميزة، الدالة الكمية وريني انتروبي. تم أيضًا أخذ التقدير من طريقة الاحتمالية القصوى لتقدير المعلمة في الاعتبار. قمنا بتوسيع ودمج توزيع ويبل العكس معه(IW) لاستكشاف السلوك المقارب للتقديرات في ظل طريقة التقدير. أدى ذلك إلى توزيع موسع جديد، والذي قمنا بعد ذلك بمحاكاته باستخدام محاكاة مونت كارلو لتقدير المعلمات غير المعروفة. وقد تبين أن توزيع NOGEIW عملي ومفيد حقًا من خلال تطبيقه على مجموعتين فعليتين من البيانات.