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Lyapunov-Schmidt reduction used to solve a nonlinear differential equation with temporal fractions

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Abstract

The present work focuses on the examination of bifurcation in periodic traveling wave solutions to a nonlinear fractional differential equation. Our methodology utilizes the Lyapunov-Schmidt reduction and He's fractional derivative techniques. An original fractional differential problem is transformed into a partial differential equation using the fractional complex transform, therefore facilitating the analysis. Consequently, we derive a simplified equation, which is formulated as a system of four nonlinear algebraic equations that align with the underlying complexity. Further, we explore the feasibility of obtaining linear approximation solutions for the nonlinear fractional differential equation.

Introduction:

A wide range of research and engineering fields are increasingly adopting fractional differential equations (FDEs). Due to their wide range of applications in several fields of applied sciences, their adaptability has attracted the interest of many academics in recent years. Finite Difference Equations (FDEs) offer a robust basis for developing models of various phenomena such as electromagnetics, solid mechanics, fluid mechanics, viscoelasticity, bio population dynamics, electrochemistry, and signal processing (referenced in [4, 5, 14]).

Given the inherent complexity of obtaining precise solutions for fractional differential equations (FDEs), several numerical methods are employed to provide efficient computing solution for addressing the wide range of FDEs. Certain fractional differential equations (FDEs) are solved using wavelet methodology, while operational matrix approaches are applied in other research. To accommodate a wider range of numerical methods suitable for various fractional differential equations (FDEs), refer to [14, 15, 17].

An important physical phenomenon, the propagation of nonlinear long waves in many environments is thoroughly examined in contexts such as oceanic dynamics, laboratory experiments on stratification, and atmospheric investigations. Several mathematical models have been developed to accurately analyse the dispersion of these nonlinear long waves from a strictly physical standpoint. A significant number of these models are derived from the widely recognized Korteweg-de Vries (KdV) equation, which is a fundamental model used to analyse weakly nonlinear long waves.

Research has shown that the KdV equation is derived by using a multiscale asymptotic method to solve the fundamental Euler equations that govern incompressible and inviscid fluids. It often refers to surface waves characterized by tiny amplitudes and long wavelengths in shallow water, as well as interior waves in fluids with shallow density stratification. The KdV equation is obtained by employing a first-order perturbation expansion that exclusively takes into account first-order dispersion and nonlinearity. Nevertheless, in numerous instances, understanding physical processes more effectively requires considering higher-order nonlinear and dispersive elements. Under such conditions, the elimination of second-order components from perturbation expansions and the application of the perturbation approach to the leading Euler equations result in the determination of a fifth-order KdV-like equation.

Operator equations are a category of nonlinear equations commonly found in disciplines such as mathematics, engineering, and physics:

$$H(x,\lambda)=0, x\in O\subset X, \lambda\in R. \qquad (1)$$

Here, $H: X\rightarrow Y$ denotes a smooth Fredholm function with a null index, Let X and Y be considered as real Banach spaces, and O defines an open set in X. The method of finite-dimensional reduction provides a solution strategy for this equation. This approach depends on using Lyapunov-Schmidt reduction to convert equation (1) into a numerically equivalent finite-dimensional equation.:

$$\Omega(\zeta,\lambda)=0, \zeta\in\widehat{M}. \qquad (2)$$

In this context, $\widehat{M}$ and $\widehat{N}$ model smooth manifolds with finite dimensions. Prior investigations [1, 4, 9, 10] have demonstrated a shift from equation (1) to equation (2) by employing a specific version of the Lyapunov-Schmidt approach. This transition guarantees that equation (2) preserves all the topological and analytic properties inherent in equation (1), including aspects such as multiplicity and bifurcation diagrams. Using the localized Lyapunov-Schmidt method, scholars such as Vainberg [5], Loginov [1], and Sapronov [8, 10] have successfully accomplished this transformation, ensuring that equation (2) maintains all the features of equation (1).

Given their substantial relevance in several fields such as accounting, economics, engineering, physics, system identification, nanofluid dynamics, fractional dynamics, and signal processing, researchers have shown a growing interest in studying fractional differential equations in recent years. Academic researchers have shifted their focus towards obtaining precise solutions for these equations by employing analytical methods such as the fractional sub-equation method [6, 7] and the first integral method [2], among others.

A study conducted in 2023 by Raza and colleagues examined the heat transport and dynamical system of nanofluids within the framework of fractional calculus. The researchers employed Riemann-Liouville and Caputo derivatives, as referenced in [14-15]. In addition, a number of scholars between 2022 and 2024 devised multiple numerical techniques to solve single and coupled systems of different forms of fractional differential equations, as referenced in [16-19].

Li and Heinlein [11, 12] A method for forming partial differential equations from fractional differential equations, therefore enabling simpler analysis using the aforementioned methods, was introduced as the fractional complex transform. This work focuses on analyzing the partitioning of periodic traveling wave solutions inside a certain nonlinear fractional differential equation. The methodology we employ combines the localized Lyapunov-Schmidt method with He's fractional derivative.

$$\frac{\partial^{\alpha}\mathcal{W}}{\partial t^{\alpha}}+5\mathcal{W}^{2}\frac{\partial^{\beta}\mathcal{W}}{\partial y^{\beta}}+5\frac{\partial^{\beta}\mathcal{W}}{\partial y^{\beta}}\frac{\partial^{2\beta}\mathcal{W}}{\partial y^{2\beta}}+5\mathcal{W}\frac{\partial^{3\beta}\mathcal{W}}{\partial y^{3\beta}}+\frac{\partial^{5\beta}\mathcal{W}}{\partial y^{5\beta}}=0\,,\quad \mathcal{W}=\mathcal{W}(y,t) \tag{3}$$

where,$\frac{\partial^{\alpha}}{\partial t^{\alpha}}$ and $\frac{\partial^{\beta}}{\partial y^{\beta}}$ are the it is a fractional derivative.and $\alpha,\beta\in(0,1]$.

Theorem1. [5]

Suppose that both X and Y denote the Banach spaces as well as $H(x,\lambda)$ represents a C^1 map given in a neighborhood U of (x_0,y_0) in the range subset in Y in a way which $\mathrm{H}(\mathrm{x}_0,\lambda_0)=0$ as well as $H_x(x_0,\lambda_0)$ represents a linear Fredholm operator. Consequently, each one of the solution sets (x,λ) of $H(x,\lambda)=0$ close to (x_0,λ_0) (with the value of λ fixed) is a one-to-one correspondence to the set of the solutions in the system with a finite dimension of N_1 of variables Finite number of variables in real equations N_0 of real variables. Moreover, $N_0=dim(Ker\ L)$ and $N_1=dim(Coker\ L)$, $(\,L=\,H_x(x_0,\lambda_0)\,)$.

Definition1. [10]

The discriminant set Σ for (1) is characterized as the collection of all values of $\lambda=\bar{\lambda}$ for which (1) shows a solution that is degenerate. $\bar{x}\in O$:

$$G(\bar{x},\bar{\lambda})=b,\qquad codim\left(Im\ \frac{\partial H}{\partial x}(\bar{x},\bar{\lambda})\right)>0$$

where, Im image represents the operator $\frac{\partial H}{\partial x}(\bar{x},\bar{\lambda})$.

Definition2. [3]

The derivative, when given as a fraction of He, may be illustrated as follows

$$D_t^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_{t_0}^{t}(s-t)^{n-\alpha-1}\left(f_0(s)-f(s)\right)ds,$$

where, $f_0(x)$ is a known function.

In order to apply the Lyapunov-Schmidt method to analyse equation (3), the first step The objective is to transform the problem into a partial differential equation. This conversion is accomplished by the fractional complex transformation defined in reference [12]:

$$Y=\frac{y^{\beta}}{\Gamma(1+\beta)}\quad ,T=\frac{t^{\alpha}}{\Gamma(1+\alpha)}\,. \tag{4}$$

The transformation of Equation (3) yields the subsequent partial differential equation:

$$\frac{\partial \mathcal{W}}{\partial T} + 5\mathcal{W}^2 \frac{\partial \mathcal{W}}{\partial y} + 5\frac{\partial \mathcal{W}}{\partial y}\frac{\partial^2 \mathcal{W}}{\partial y^2} + 5\mathcal{W}\frac{\partial^3 \mathcal{W}}{\partial y^3} + \frac{\partial^5 \mathcal{W}}{\partial y^5} = 0\,, \quad \mathcal{W} = \mathcal{W}(y,t) \qquad (5)$$

To analyze the solutions of equation (5) associated with moving waves, we employ the following transformation.

$$\mathcal{W}(y,t) = u(x) = u(k_1 Y - k_2 T)\ , \quad k_1 = c_1\beta\,, k_2 = c_2\alpha \qquad (6)$$

Where c_1, c_2 are real constants and α, β are parameters. Substituting equation (6) into equation (5) reduces equation (5) to a fourth-order nonlinear ordinary differential equation (ODE).

$$k_1^4 u'''' + 5k_1^2 uu'' + \frac{5}{3}u^3 - k_2 u = 0, \quad ' = d/dx \qquad (7)$$

The present work establishes u as a periodic function, represented by $u(x) = u(x + 2\pi)$. In the following part, we will use the localized Lyapunov-Schmidt approach to simplify equation (7) into a defined set of nonlinear algebraic equations that are finite-dimensional and can be compared.

Bifurcation Equation Simplification (Reduction Technique)

The objective is to examine the bifurcation of moving wave solutions that are periodic to (5), it's advantageous to express (7) as an operator equation.

$$H(u,\lambda) = k_1^4 u'''' + 5k_1^2 uu'' + \frac{5}{3}u^3 - k_2 u. \qquad (8)$$

Here, $H: E \to M$ represents a nonlinear Fredholm operator with zero index, where $E = \Pi_4([0,2\pi], \mathbb{R})$ denotes the space containing each function having derivatives of order up to 4, and $M = \Pi_0([0,2\pi], \mathbb{R})$ represents all continuous functions that are periodic comprise the space. Here, $\mathbb{R}$ denotes the real space, and $u = u(x)$, where $x \in [0, 2\pi]$. Hence, the solution for bifurcation specified in equation (8) precisely corresponds to the solution of the operator equation.

$$H(u,\lambda) = 0. \qquad (9)$$

According to Theorem (1), the solutions of equation (5) are interchangeable with a system of finite dimensions solved comprising 2 variables and 2 equations, where 2 represents the dimension of both $kerH_u(0,\lambda)$ and $co - kerH_u(0,\lambda)$. The initial step in this simplification technique is to obtain the linearized equation that corresponds to equation (9), denoted as:

$$Ah = 0, \quad h \in E$$

$$A = \frac{\partial F}{\partial u}(0,\lambda) = k_1^4 \frac{d^4}{dx^4} - k_2.$$

An expression for the periodic solution of the linearized equation is provided.

$$e_p(x) = a_p\, sin(px) + b_p\, cos(px)\ , \quad p = 1,2,3,\cdots$$

The characteristic equation matching this answer is

$$k_1^4 p^4 - k_2 = 0$$

Induce bifurcation into four distinct modes

$e_1 = a_1 \, sin(x), \quad e_2 = a_2 \, cos(x) , e_3 = a_3 \, sin(2x), \quad e_4 = a_4 \, cos(2x)$.

Where $\|e_i\|_{\mathcal{H}} = 1$ and $a_i = \sqrt{2}$ for $i = 1, 2,3,4$ ($\mathcal{H}$ is Hilbert space $L^2([0,2\pi], R)$). Let $N = Ker(A) = span\{e_1 , e_2, e_3 , e_4\}$, Let the space E be divided into four subspaces by taking the direct sum of, N and $N^{\perp}$ which is the orthogonal complement to N,

$E = N \oplus N^{\perp} , N^{\perp} = \{v \in E: v \perp N\}$

Similarly, may be expressed as the direct sum of two subspaces, $\widehat{N}$ and $\widehat{N}^{\perp}$ which are orthogonal complements of $\widehat{N}$,

$M = \widehat{N} \oplus \widehat{N}^{\perp} \; , \qquad \widehat{N}^{\perp} = \{\omega \in M: \omega \perp \widehat{N}\}$

The breakdown of expression E indicated above implies the existence of two projections, $P: E \to N$ and $I - P: E \to N^{\perp}$ where (The identity operator is denoted as I),

$Pu = \mathcal{W}, \quad (I - P)u = v,$

Consequently, each element $y \in E$ possesses a distinct and exclusive form,

$u = z + v \, , \;\; z = \sum_{i=1}^{4} \zeta_i e_i \in N \, , \;\; v \in N^{\perp} , \zeta_i = \langle u, e_i \rangle$.

Let $\langle \cdot,\cdot \rangle$ be inner products in Hilbert space $L^2(\, [\, 0, \pi \,] \, , \mathbb{R})$.
Furthermore, the breakdown of matrix M indicates the existence of two projections $Q: M \to \widehat{N}$ and $I - Q: M \to \widehat{N}^{\perp}$ such that

$QH(u, \lambda) = H_1(u, \lambda), \quad (I - Q)H(u, \lambda) = H_2(u, \lambda),$

consequently, any element $H(u, \lambda) \in M$ can be expressed in a distinct manner,

$$\begin{aligned} H(u, \lambda) &= H_1(u, \lambda) + H_2(u, \lambda) \\ &= QH(u, \lambda) + (I - Q)H(u, \lambda) = 0 \end{aligned}$$

$$H_1(u, \lambda) = \sum_{i=1}^{4} v_i(u, \lambda) e_i \in N \, , \quad H_2(u, \lambda) \in \widehat{N}^{\perp} \, ,$$

$v_i(u, \lambda) = \langle H(u, \lambda), e_i \rangle_{\mathcal{H}}$.

Accordingly, equation (9) is transformed as,

$QH(u, \lambda) = 0 \, , \;\; (I - Q)H(u, \lambda) = 0.$
or
$QH(z + v, \lambda) = 0 \;\; , \;\; (I - Q)H(z + v, \lambda) = 0.$

By the implicit function theorem, a smooth map $\varphi: N \to N^{\perp}$ such that $\varphi(u, \lambda) = v$ and $(I - Q)H(z + \varphi(u, \lambda), \lambda) = 0.$

The solution to equation (9) within the vicinity of the point $u = 0$ can be obtained by solving the following equation:

$$QH(u + \varphi(u, \lambda), \lambda) = 0. \qquad (10)$$

The solutions to equation (10) exhibit bifurcation and have the following form,

$\phi(\zeta, q) = 0, \zeta = (\zeta_1, \zeta_2)$

where

$\phi(\zeta, \lambda) = H_1(z + \varphi(u, \lambda), \lambda).$

Mathematical equation (9) may be expressed as,

$H(z + v, \lambda) = A(z + v) + T(z + v) = Az + \omega(z) + \cdots,$

$\omega(z) = 5zz'' + \frac{5}{3}z^3,$

where $T(z + v) = \omega(z + v)$ and The dots represent the fractional terms of element v. Thus,

$$\phi(\zeta, q, k_1) = H_1(z + v, \lambda) = \sum_{i=1}^{4} \langle Az + \omega(z), e_i \rangle_{\mathcal{H}} e_i + \cdots = 0 \qquad (11)$$

after some calculations of equation (11) we have the following system

$$\phi(\zeta, q, k_1) = \begin{pmatrix} q\zeta_1 + \frac{25\sqrt{2}}{2}k_1^2(\zeta_1\zeta_4 - \zeta_2\zeta_3) + \frac{5}{2}k_1^2\zeta_1(\zeta_1^2 + \zeta_2^2 + 2\zeta_3^2 + 2\zeta_4^2) \\ q\zeta_2 - \frac{25\sqrt{2}}{2}k_1^2(\zeta_1\zeta_3 + \zeta_2\zeta_4) + \frac{5}{2}k_1^2\zeta_2(\zeta_1^2 + \zeta_2^2 + 2\zeta_3^2 + 2\zeta_4^2) \\ (15 + q)\zeta_3 - 5\sqrt{2}k_1^2\zeta_1\zeta_2 + \frac{5}{2}k_1^2\zeta_3(2\zeta_1^2 + 2\zeta_2^2 + \zeta_3^2 + \zeta_4^2) \\ (15 + q)\zeta_4 + \frac{5\sqrt{2}}{2}k_1^2(\zeta_1^2 - \zeta_2^2) + \frac{5}{2}k_1^2\zeta_4(2\zeta_1^2 + 2\zeta_2^2 + \zeta_3^2 + \zeta_4^2) \end{pmatrix} = 0 \qquad (12)$$

Where $q = 1 - k_1$ and $\langle \omega(z), h(x) \rangle_{\mathcal{H}} = \int_0^{2\pi} \omega(x)h(x)dx$.

Bifurcation Analysis of system (12)

By changing variables $\eta_i = \sqrt[3]{\frac{5k_1^2}{2}}\zeta_i$, $i = 1, \ldots, 4$ We have obtained system (12) which is analogous to the subsequent system,

$$\phi(\hat{\eta}, \varepsilon) = \begin{pmatrix} \alpha_1\eta_1 + \alpha_2(\eta_1\eta_4 - \eta_2\eta_3) + \eta_1(\eta_1^2 + \eta_2^2 + 2\eta_3^2 + 2\eta_4^2) \\ \alpha_1\eta_2 - \alpha_2(\eta_1\eta_3 + \eta_2\eta_4) + \eta_2(\eta_1^2 + \eta_2^2 + 2\eta_3^2 + 2\eta_4^2) \\ \alpha_3\eta_3 - \alpha_4\eta_1\eta_2 + \eta_3(2\eta_1^2 + 2\eta_2^2 + \eta_3^2 + \eta_4^2) \\ \alpha_3\eta_4 + \alpha_5(\eta_1^2 - \eta_2^2) + \eta_4(2\eta_1^2 + 2\eta_2^2 + \eta_3^2 + \eta_4^2) \end{pmatrix}$$

Where, $\hat{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4)$, and $\varepsilon = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$.

In the complex variables,

$$z_1 = \eta_1 + i\eta_2 \ , \ z_2 = \eta_3 + i\eta_4 \quad , \quad i = \sqrt{-1}$$

The bifurcation equation may be expressed as follow:

$$G(\tilde{z}, \varepsilon) = \begin{pmatrix} G_1(z, \varepsilon) \\ G_2(z, \varepsilon) \end{pmatrix} = 0 \ , \tilde{z} = (z_1, z_2)$$

Where,

$G_1(\tilde{z}, \varepsilon) = \phi_1(\hat{\eta}, \varepsilon) + i\phi_2(\hat{\eta}, \varepsilon) = 0$

$G_2(\tilde{z},\varepsilon) = \phi_3(\hat{\eta},\varepsilon) + i\phi_4(\hat{\eta},\varepsilon) = 0$

And

$$\phi_1(\hat{\eta},\varepsilon) = \alpha_1\eta_1 + \alpha_2(\eta_1\eta_4 - \eta_2\eta_3) + \eta_1(\eta_1^2 + \eta_2^2 + 2\eta_3^2 + 2\eta_4^2)$$
$$\phi_2(\hat{\eta},\varepsilon) = \alpha_1\eta_2 - \alpha_2(\eta_1\eta_3 + \eta_2\eta_4) + \eta_2(\eta_1^2 + \eta_2^2 + 2\eta_3^2 + 2\eta_4^2)$$
$$\phi_3(\hat{\eta},\varepsilon) = \alpha_3\eta_4 - \alpha_4\eta_1\eta_2 + \eta_3(2\eta_1^2 + 2\eta_2^2 + \eta_3^2 + \eta_4^2)$$
$$\phi_4(\hat{\eta},\varepsilon) = \alpha_3\eta_4 + \alpha_5(\eta_1^2 - \eta_2^2) + \eta_4(2\eta_1^2 + 2\eta_2^2 + \eta_3^2 + \eta_4^2)$$

Therefore, we have two complex equations in the following form,

$$\begin{cases} Z_1|Z_1|^2 + 2Z_1|Z_2|^2 + \alpha_1 Z_1 - i\alpha_2 \bar{Z}_1 Z_2 = 0 \\ Z_2|Z_2|^2 + 2Z_2|Z_1|^2 + \alpha_3 Z_1 - i\alpha_5 Z_1^2 = 0 \end{cases} \tag{13}$$

Where,

$|Z_1|^2 = \eta_1^2 + \eta_2^2$, $|Z_2|^2 = \eta_3^2 + \eta_4^2$, $\bar{Z}_1$ is a conjugate of Z_1.

The solution to system (13) of complex equations may be conveniently obtained by relocating to the polar coordinate, by $Z_1 = r_1 e^{i\theta_1}$ and $Z_2 = r_2 e^{i\theta_2}$

In the real component of the polar coordinate system, we possess a system of two equations.

$$\begin{cases} r_1^3 + 2r_1 r_2^2 + \alpha_1 r_1 - \alpha_2 r_1 r_2 \sin(2\theta_1 - \theta_2) = 0 \\ r_2^3 + 2r_1^2 r_2 + c r_2 + \alpha_5 r_1^2 \sin(2\theta_1 - \theta_2) = 0 \end{cases} \tag{14}$$

Where, $a = \alpha_1$, $c = \alpha_3$, $b = \alpha_2 \sin(2\theta_1 - \theta_2)$, $d = \alpha_5 \sin(2\theta_1 - \theta_2)$

If $b = \frac{5}{2}$, $d = \frac{1}{2}$, we have $\theta_1 = \theta_2 = \frac{\pi}{4}$

Furthermore, Within the real component of the polar coordinate system, we possess a system of two equations.

$$\begin{cases} r_1^3 + 2r_1 r_2^2 + \alpha_1 r_1 - \frac{5}{2} r_1 r_2 = 0 \\ r_2^3 + 2r_1^2 r_2 + c r_2 + \frac{1}{2} r_1^2 = 0 \end{cases} \tag{15}$$

The set of singular points for the polar system (15) may be expressed in the following manner:

$$r_1^2 r_2 - \frac{15}{2} r_2^3 - \frac{5}{2} c r_2 + 6r_1^4 - 3r_1^2 r_2^2 + 3cr_1^2 + 6r_2^4 + 2cr_2^2 + 2ar_1^2 + 3ar_2^2 + ac + \frac{5}{2} r_1^2 = 0$$

To study the discriminant set of system (15) of polar equations and determine all the segments of the discriminant set defined in the alternating current plane.

We shall now outline the discriminant set for equation (15) in the $ac-plane$ with the regular solutions in the all region in the following figure.

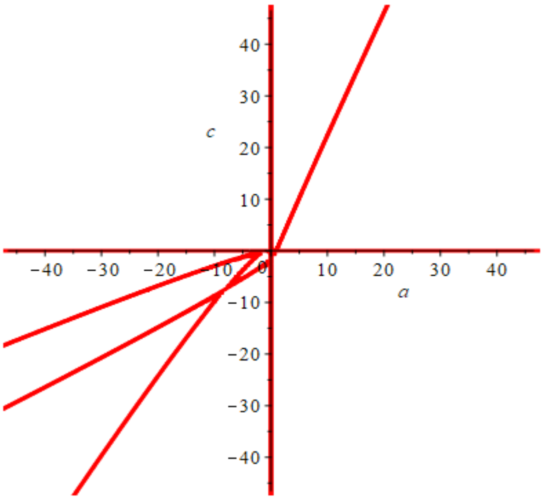

Fig 1. *the discriminant set of system (15)*

We choose $a = -5$, $c = -30$ There are five genuine solutions of system (15) for these data.

$P_1 = (2.303217307, 0.1369221194)$

$P_2 = (-2.303217307, 0 .1369221194)$

$P_3 = (\ 0\ , 5.477225575)$

$P_4 = (\ 0\ , -5.477225575)$

$P_5 = (\ 0\ , 0)$

According to the above parameters, the fractional solutions of the equation can be represented by graphs that provide a linear approximation.

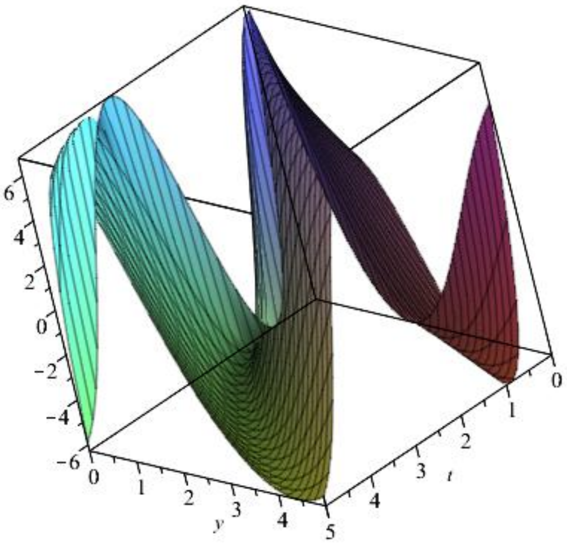

Fig 2. *Geometrical representation of* P_1

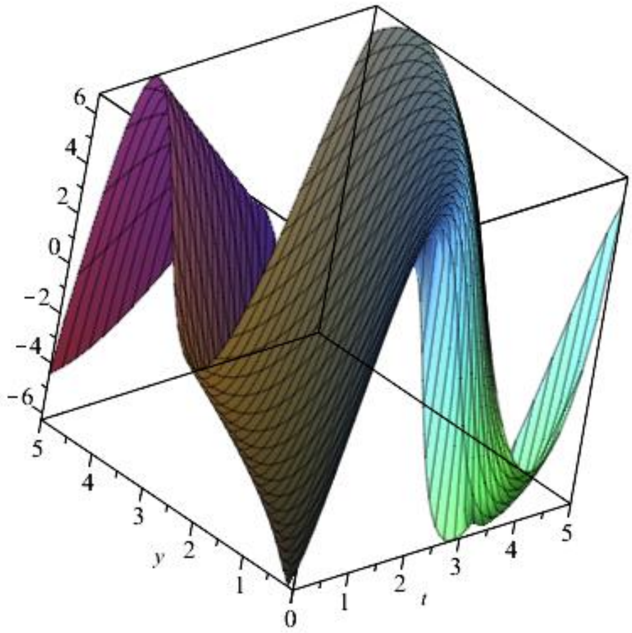

Fig 3. *Geometrical representation of* P_2

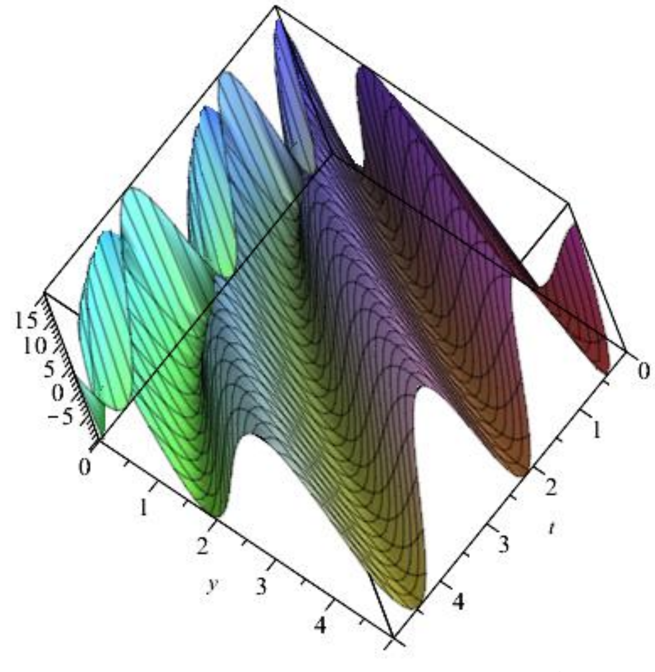

Fig 4. *Geometrical representation of* P_3

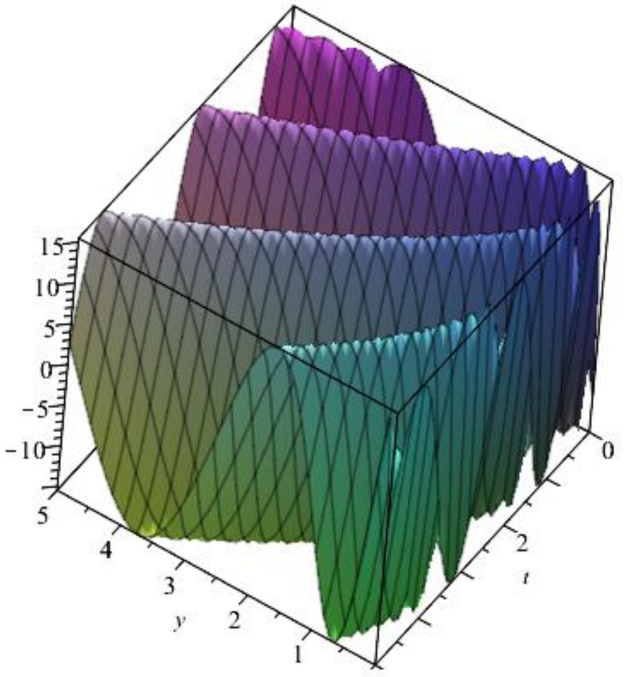

Fig 5. *Geometrical representation of* P_4

CONCLUSION

The present study investigates the phenomenon of bifurcation in periodic travelling wave solutions to non-linear fractional differential equations by the utilization of He's fractional derivative and the Lyapunov-Schmidt reduction. To convert a fractional differential equation into a partial differential equation, the fractional complex transform is employed. This partial differential equation is then reduced to an ordinary differential equation (ODE) using the travelling wave transformation. The simplified ordinary differential equation (ODE) has been determined as a set of four nonlinear algebraic equations. We presented systematic linear approximations to the nonlinear fractional differential equation using geometric methods. Lastly, we demonstrate the application of topological approaches in the analysis of nonlinear fractional differential equations.

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استخدام طريقة اختزال ليابونوف-شميدت لحل معادلة تفاضلية غير خطية ذات كسور زمنية

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الخلاصة:

يركز العمل الحالي على فحص التشعب في حلول الموجات المتنقلة الدورية لمعادلة تفاضلية كسرية غير خطية. نستخدم طرق اختزال ليابونوف-شميدت والمشتقة الكسرية لهي. يتم استخدام التحويل المركب الكسري لتحويل المشكلة التفاضلية الكسرية الأولية إلى معادلة تفاضلية جزئية، وبالتالي تسهيل التحليل. وبالتالي، نستنتج معادلة مبسطة، والتي تمت صياغتها كمجموعة من أربع معادلات جبرية غير خطية تتوافق مع المشكلة الأساسية. علاوة على ذلك، نستكشف جدوى الحصول على حلول تقريب خطية لمعادلة التفاضلية الكسرية غير الخطية.