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Iterative algorithms for a class of implicit quasi variational inequalities

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Abstract

It is known that one of the most paramount and complicated problems in variational inequality Theory (VIT) is the development of an efficacious and implementable approximation schemes for solving diverse categories of variational inequality. Thus, through this study we introduce a new iterative scheme with errors to solve generalized strongly nonlinear implicit quasivariational inequalities (GSNIQVI) by fixed point defined on for a set-valued mapping $\emptyset \neq \mathfrak{D} \subset \Xi, \mathfrak{D} : \Xi \longrightarrow 2^{\Xi}$ with closed convex values in Hilbert space Ξ. These iterative schemes are constructed for Lipschitz continuous mappings, Γ-strongly monotone, and Θ-strongly monotone mappings under some appropriate conditions. These iterative schemes is solving special case of GSNIQVI with errors and without errors. We proved an existence and uniqueness results for solution GNIQVIP and convergence of new iterative algorithms with errors, and without errors, for several mappings: the first $\Gamma: \Xi \longrightarrow \Xi$ is a Lipschitz continuous mapping, the second $\hat{A}, \hat{C}, \hat{E}, \hat{U}: \Xi \longrightarrow \Xi$ are Lipschitz continuous mappings, the third $\check{N}: \Xi^4 \longrightarrow \Xi$ is a mapping such that $\check{N}(\hat{A}, \hat{C}, \hat{E}, \hat{U})$ is a strongly monotone with respect to $\hat{A}$ in $\check{N}_1$, Γ-strongly monotone with respect to $\hat{C}$ in $\check{N}_2$, Θ-strongly monotone with respect to $\hat{E}$ in $\check{N}_3$ and Lipschitz continuous with respect to $\check{N}_1, \check{N}_2, \check{N}_3, \check{N}_4$ and the fourth $P_{\mathfrak{D}}(\breve{g}) : \mathfrak{D} \rightarrow \Xi$ is the metric projection which is Lipschitz continuous. Our results is inspired and encouraged for several research works in the literature.

Introduction and Preliminaries

The well-known that variational inequality (VI) theory play a remarkable and major role in the etude of a fluid flow through porous media, nonlinear programming, economics and transportation equilibrium, physics, contact problems in elasticity, mechanics, optimization and control, and many another offshoots of engineering sciences and mathematical.

Over many years the researchers arising a several new iterative algorithm which contains some known algorithm as particular status for approximation of solutions GNIQVIP and GNIQV inclusion problem which include VI, QVI and variational inclusions , quasi-variational inclusions, and also proved the convergence of them, (for example see [1-9]).

Inspired and encouraged by recent research works [1], [3], [8, 9] and other reference in literature, we prove an existence and uniqueness theorems for solution of GNIQVIP and convergence of new iterative algorithms with errors and without errors, including Lipschitz continuous, strongly monotone mappings in Hilbert spaces.

Before we recall and introduce the important preliminaries, using the following symbols:

- Ξ ≔ real Hilbert space,
- $\overline{\mathfrak{D}}$ ≔ Set-valued mapping with nonempty closed convex values subset of Ξ,
- $\tilde{\mathrm{I}}$ the identity mapping of Ξ.
- $\mathcal{F}(\widetilde{\mathcal{K}})$ ≔ The set of all fixed points of $\widetilde{\mathcal{K}}$.
  - GNQVIP ≔ Ggeneralized nonlinear quasivariational inequalities problem
  - GNIQVIP ≔ Generalized nonlinear implicit quasivariational inequalities problem
  - $\check{\mathrm{N}}_1$≔ The first argument of $\check{\mathrm{N}}$
  - $\check{\mathrm{N}}_2$≔ The second argument of $\check{\mathrm{N}}$
  - $\check{\mathrm{N}}_3$≔ The third argument of $\check{\mathrm{N}}$
  - $\check{\mathrm{N}}_4$≔ The fourth argument of $\check{\mathrm{N}}$

In this work á, ć, é, â, ĉ, ê, û, ƀ, ǰ, ħ, ḟ, ђ, τ, γ, and ž are positive constants.

Consider space Ξ, with inner product and norm denoted by $\langle .,. \rangle$ and $\| . \|$, respectively, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping with nonempty closed convex values. Let $\hat{\mathrm{A}}, \hat{\mathrm{C}}, \hat{\mathrm{E}}, \hat{\mathrm{U}}, \Gamma: \Xi \longrightarrow \Xi$ be mapping, and $\acute{m}: \Xi \longrightarrow \Xi$ is mapping, $\mathrm{P}_{\overline{\mathfrak{D}}} : \overline{\mathfrak{D}} \to \Xi$ is the metric projection which assigns, to each $\breve{g} \in \Xi$, the unique point in $\overline{\mathfrak{D}}$, denoted $\mathrm{P}_{\overline{\mathfrak{D}}}(\breve{g})$, such that

$$\|\breve{g} - \mathrm{P}_{\overline{\mathfrak{D}}}(\breve{g})\| = \inf\{\|\breve{g} - \breve{y}\| : \forall \breve{y} \in \overline{\mathfrak{D}}\} \tag{1}$$

Now, define collection of mappings $\check{\mathrm{N}}: \Xi^4 \to \Xi$ such that $\check{\mathrm{N}}\left(\hat{\mathrm{A}}, \hat{\mathrm{C}}, \hat{\mathrm{E}}, \hat{\mathrm{U}}\right)$ is:

- á-strongly monotone with respect to $\hat{\mathrm{A}}$ $\check{\mathrm{N}}_1$, with positive constant á ,
- Γ-strongly monotone with respect to $\hat{\mathrm{C}}$ in $\check{\mathrm{N}}_2$, with positive constant ć . (2)
- ϴ-strongly monotone with respect to $\hat{\mathrm{E}}$ in $\check{\mathrm{N}}_3$, with positive constant é ,
- Lipschitz continuous with respect to $\check{\mathrm{N}}_1, \check{\mathrm{N}}_2, \check{\mathrm{N}}_3$ and $\check{\mathrm{N}}_4$ with positive constants â, ĉ, ê, û, respectively.

We deem the next problem: Find $\breve{g} \in \Xi$, such that $\Gamma(\breve{g}) \in \overline{\mathfrak{D}}(\breve{g})$

$$\langle \Gamma(\breve{g}) - \check{\mathrm{N}}(\hat{\mathrm{A}}\breve{g}, \hat{\mathrm{C}}\breve{g}, \hat{\mathrm{E}}\breve{g}, \hat{\mathrm{U}}\breve{g}), \breve{y} - \Gamma(\breve{g}) \rangle \geq 0, \forall \breve{y} \in \overline{\mathfrak{D}}(\breve{g}) \tag{3}$$

The problem (3) is called GSNIQVIP.

Remark 1.1:

If $\overline{\mathfrak{D}}(\breve{g}) = \acute{m}(\breve{g}) + \overline{\mathfrak{D}}$, $\emptyset \neq \overline{\mathfrak{D}} \subset \Xi$. then the problem (3) is equivalent to finding $\breve{g} \in \Xi$ Such that $\Gamma(\breve{g}) - \acute{m}(\breve{g}) \in \overline{\mathfrak{D}}$

$$\langle \Gamma(\breve{g}) - \check{\mathrm{N}}(\hat{\mathrm{A}}\breve{g}, \hat{\mathrm{C}}\breve{g}, \hat{\mathrm{E}}\breve{g}, \hat{\mathrm{U}}\breve{g}), \breve{y} - \Gamma(\breve{g}) \rangle \geq 0, \forall \breve{y} \in \acute{m}(\breve{g}) + \overline{\mathfrak{D}} \tag{4}$$

is called GNQVIP.

Assume that $\overline{\mathfrak{D}}$ closed and convex and $\emptyset \neq \overline{\mathfrak{D}} \subseteq \Xi$. A mapping : $\overline{\mathfrak{D}} \to \Xi$ is said to be

i. **Strongly monotone** [10, 11] if there exists a constant $\tau > 0$ which in

$$\langle \widetilde{\mathcal{K}}\breve{g} - \widetilde{\mathcal{K}}t, \breve{g} - \breve{y}\rangle \geq \tau \|\breve{g} - \breve{y}\|^2, \ \forall \breve{g}, \breve{y} \in \mathfrak{D},$$

ii. **Monotone** [11] if

$$\langle \widetilde{\mathcal{K}}\breve{g} - \widetilde{\mathcal{K}}t, \breve{g} - \breve{y}\rangle \geq 0, \ \forall \breve{g}, \breve{y} \in \mathfrak{D},$$

iii. **Lipschitz continuous** [12] if there exists a constant η > 0 which in

$$\|\widetilde{\mathcal{K}}\breve{g} - \widetilde{\mathcal{K}}t\| \leq \eta \|\breve{g} - \breve{y}\|, \forall \breve{g}, \breve{y} \in \overline{\mathfrak{D}}.$$

In the following, recall the needed lemmas:

Lemma 1.2. [13] Let $\{ℸ_n\}, \{\varepsilon_n\}$ and $\{\sigma_n\}$ be sequences of nonnegative real numbers such that

$$ℸ_{n+1} \leq (1 - \Delta_n)ℸ_n + \varepsilon_n + \sigma_n, \ n \geq 0.$$

where $\{\Delta_n\} \in [0, 1], \sum_{n=0}^{\infty} \Delta_n = \infty, \ , \varepsilon_n = O(\Delta_n), \sum_{n=1}^{\infty} \sigma_n < \infty$.

Then, $\lim_{n\to\infty} ℸ_n = 0$.

Lemma 1.3. [14] If $\breve{g} \in \Xi$ and $\breve{y} \in \overline{\mathfrak{D}}$, then $\breve{g} = P_{\overline{\mathfrak{D}}}(\breve{y})$ if and only if the next inequality satisfies:

$$\langle \breve{g} - \breve{y}, t^{\#} - \breve{g}\rangle \geq 0, \ \forall t^{\#} \in \overline{\mathfrak{D}}.$$

Lemma 1.4. [15] The mapping $P_{\overline{\mathfrak{D}}}$ defined by (1) is non-expansive, i.e

$$\|P_{\overline{\mathfrak{D}}}(\breve{g}) - P_{\overline{\mathfrak{D}}}(\breve{y})\| \leq \|\breve{g} - \breve{y}\|, \forall \breve{g}, \breve{y} \in \overline{\mathfrak{D}}.$$

Lemma 1.5. [15] If $\overline{\mathfrak{D}}(\breve{g}) = \acute{m}(\breve{g}) + \overline{\mathfrak{D}}$, then $\forall \breve{g}, \breve{y} \in \Xi$, we yield

$$P_{\overline{\mathfrak{D}}(\breve{g})}(\breve{y}) = \acute{m}(\breve{g}) + P_{\overline{\mathfrak{D}}}(\breve{y} - \acute{m}(\breve{g}))$$

If there exists a point $\breve{g} \in \overline{\mathfrak{D}}$ such that $\breve{g} = \widetilde{\mathcal{K}}\breve{g}$, then $\breve{g}$ is called be fixed point of $\widetilde{\mathcal{K}}$. It is well famous that $\mathcal{F}(\widetilde{\mathcal{K}})$ is closed and convex if $\mathcal{K}$ is nonexpansive. There are many Authors studied a fixed point as [16-19] , also many applications and related papers for this field such as [20-23].

From Lemma (1.3) and (1.5), it is well-known that the QNIVIP in (2) has a unique solution if and only if the mapping $\tilde{O}: \Xi \longrightarrow \Xi$

$$\tilde{O}(\breve{g}) = \breve{g} - \Gamma(\breve{g}) + P_{\overline{\mathfrak{D}}(\breve{g})}\left[\Gamma(\breve{g}) - ƀ\left(\Gamma(\breve{g}) - \check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g})\right)\right] \quad (5)$$

has a unique fixed point, $ƀ > 0$.

2. Main Results

Important new definitions and proposition are presented below:

Definition 2.1: Suppose that $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping s.t $\forall \breve{g} \in \Xi, \ \emptyset \neq \overline{\mathfrak{D}} \subseteq \Xi$ is closed convex. The projection $P_{\overline{\mathfrak{D}}}(\breve{g})$ is ṕ-Lipschitz continuous, if ∃ ṕ such that

$$\|P_{\overline{\mathfrak{D}}(\breve{g})}(\hat{s}) - P_{\overline{\mathfrak{D}}(\breve{y})}(\hat{s})\| \leq ṕ\|\breve{g} - \breve{y}\|, \qquad \forall \breve{g}, \breve{y}, \check{z} \in \Xi.$$

Definition 2.2: Suppose that $\hat{A}: \Xi \longrightarrow \Xi$ and $\check{N}: \Xi^4 \rightarrow \Xi$ be mappings.

The mapping Ň is called:

• Strongly monotone with respect to Â in $\check{N}_1$, if ∃ á > 0 such that

$\langle \check{N}(\hat{A}\breve{g},.,.,.) - \check{N}(\hat{A}\breve{y},.,.,.,.), \breve{g} - \breve{y} \rangle \geq á\|\breve{g} - \breve{y}\|^2 \;\; \forall \breve{g}, \breve{y} \in \Xi$.

• Lipschitz continuous with respect to $\check{N}_1$ if ∃ â > 0 such that

$\|\check{N}(\breve{g},.,.,.) - \check{N}(\breve{y},.,.,.,.)\| \leq â\|\breve{g} - \breve{y}\|, \;\forall \breve{g}, \breve{y}, \in \Xi.$

By the same way, we can define strongly monotone and Lipschitz continuity of Ň with respect to $\check{N}_2, \check{N}_3$, and $\check{N}_4$

• Â is Lipschitz continuous if ∃ ǰ > 0 such that

$\|\hat{A}\breve{g} - \hat{A}\,\breve{y}\| \leq ǰ\|\breve{g} - \breve{y}\|, \;\forall \breve{g}, \breve{y}, \in \Xi$

Definition 2.3. Assume that Ĉ, Ê, Γ, Ɵ: Ξ ⟶ Ξ be mappings. The mapping $\check{N}: \Xi^4 \to \Xi$ is called:

• Γ-strongly monotone with respect to $\check{N}_2$, if

$\langle \check{N}(.,\hat{C}\breve{g},.,.) - \check{N}(.,\hat{C}\breve{y},.,.), \Gamma(\breve{g}) - \Gamma(\breve{y}) \rangle \geq ć \;\|\Gamma(\breve{g}) - \Gamma(\breve{y})\|^2 \;\; \forall \breve{g}, \breve{y} \in \Xi, ć > 0$

• Ɵ-strongly monotone with respect to $\check{N}_3$, if

$$\langle \check{N}(.,.,\hat{E}\breve{g},.) - \check{N}(.,.,\hat{E}\breve{y},.), \Gamma(\breve{g}) - \Gamma(\breve{y}) \rangle \geq é \;\|Ɵ(\breve{g}) - Ɵ(\breve{y})\|^2 \;\; \forall \breve{g}, \breve{y} \in \Xi, é > 0$$

Algorithm 2.4. Assume that $\emptyset \neq \overline{\mathfrak{D}} \subset \Xi$, $\overline{\mathfrak{D}}: \Xi \longrightarrow 2^{\Xi}$ is a closed convex set-valued mapping, Let Â, Ĉ, Ê, Û, Γ: Ξ ⟶ Ξ be mappings, $\check{N}: \Xi^4 \to \Xi$ be a mapping defined by (2), and $P_{\overline{\mathfrak{D}}}(\breve{g})$ is ṕ-Lipschitz continuous mapping. For arbitrary $\breve{g}_0 \in \overline{\mathfrak{D}}$, define a sequence $\{\breve{g}_n\}$ with errors as follows:

$$\breve{g}_{n+1} = \breve{y}_n - \Gamma(\breve{y}_n) + P_{\overline{\mathfrak{D}}}(\breve{y}_n)\left[\Gamma(\breve{y}_n) - ƀ\left(\Gamma(\breve{y}_n) - \check{N}(\hat{A}\breve{y}_n, \hat{C}\breve{y}_n, \hat{E}\breve{y}_n, \hat{U}\breve{y}_n)\right)\right] + \eta_n ú_n,$$

$$\breve{y}_n = ά_n\left\{\breve{g}_n - \Gamma(\breve{g}_n) + P_{\overline{\mathfrak{D}}}(\breve{g}_n)\left[\Gamma(\breve{g}_n) - ƀ\left(\Gamma(\breve{g}_n) - \check{N}(\hat{A}\breve{g}_n, \hat{C}\breve{g}_n, \hat{E}\breve{g}_n, \hat{U}\breve{g}_n)\right)\right]\right\}$$

$$+ έ_n\left\{ȶ_n - \Gamma(ȶ_n) + P_{\overline{\mathfrak{D}}}(ȶ_n)\left[\Gamma(ȶ_n) - ƀ\left(\Gamma(ȶ_n) - \check{N}(\hat{A}ȶ_n, \hat{C}ȶ_n, \hat{E}ȶ_n, \hat{U}ȶ_n)\right)\right]\right\} + ή_n v́_n,$$

$$ȶ_n = α̋_n\breve{g}_n + ε̋_n\left\{\breve{g}_n - \Gamma(\breve{g}_n) + P_{\overline{\mathfrak{D}}}(\breve{g}_n)\left[\Gamma(\breve{g}_n) - ƀ\left(\Gamma(\breve{g}_n) - \check{N}(\hat{A}\breve{g}_n, \hat{C}\breve{g}_n, \hat{E}\breve{g}_n, \hat{U}\breve{g}_n)\right)\right]\right\}$$
$$+ η̋_n ẃ_n, n \geq 0. \qquad (6)$$

where $\{ú_n\}, \{v́_n\}, \{ẃ_n\}$ be bounded sequences in Ξ, and $\{\eta_n\}, \{ά_n\}, \{έ_n\}, \{ή_n\}, \{α̋_n\}, \{ε̋_n\}, \{η̋_n\}$ are sequences in [0, 1] holding the next conditions:

- $ά_n + έ_n + ή_n = 1, α̋_n + ε̋_n + η̋_n = 1, n \geq 0,$ (7)

- $\lim_{n\to\infty} \eta_n = \lim_{n\to\infty} ή_n = \lim_{n\to\infty} η̋_n = 0,\; \sum_{n=1}^{\infty} έ_n = \infty; \sum_{n=1}^{\infty} \eta_n < \infty;$ and $\sum_{n=1}^{\infty} ή_n < \infty.$ (8)

Algorithm 2.5: Assume that $\emptyset \neq \overline{\mathfrak{D}} \subset \Xi$, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a closed convex set-valued mapping, Let Â, Ĉ, Ê, Û, Γ: Ξ ⟶ Ξ be mappings, Ň: $\Xi^4 \rightarrow \Xi$ be a mapping defined by (2), and $P_{\overline{\mathfrak{D}}}(\breve{g})$ is ṕ-Lipschitz continuous mapping. For arbitrary $\breve{g}_0 \in \overline{\mathfrak{D}}$, define a sequence $\{\breve{g}_n\}$ as follows:

$$\breve{g}_{n+1} = \breve{y}_n - \Gamma(\breve{y}_n) + P_{\overline{\mathfrak{D}}}(\breve{y}_n)\left[\Gamma(\breve{y}_n) - ƀ\left(\Gamma(\breve{y}_n) - \check{N}(\hat{A}\breve{y}_n, \hat{C}\breve{y}_n, \hat{E}\breve{y}_n, \hat{U}\breve{y}_n)\right)\right],$$

$$\breve{y}_n = \acute{\alpha}_n\left\{\breve{g}_n - \Gamma(\breve{g}_n) + P_{\overline{\mathfrak{D}}}(\breve{g}_n)\left[\Gamma(\breve{g}_n) - ƀ\left(\Gamma(\breve{g}_n) - \check{N}(\hat{A}\breve{g}_n, \hat{C}\breve{g}_n, \hat{E}\breve{g}_n, \hat{U}\breve{g}_n)\right)\right]\right\}$$

$$+\acute{\varepsilon}_n\left\{t_n - \Gamma(t_n) + P_{\overline{\mathfrak{D}}}(t_n)\left[\Gamma(t_n) - ƀ\left(\Gamma(t_n) - \check{N}(\hat{A}t_n, \hat{C}t_n, \hat{E}t_n, \hat{U}t_n)\right)\right]\right\},$$

$$t_n = \ddot{\alpha}_n\breve{g}_n + \ddot{\varepsilon}_n\left\{\breve{g}_n - \Gamma(\breve{g}_n) + P_{\overline{\mathfrak{D}}}(\breve{g}_n)\left[\Gamma(\breve{g}_n) - ƀ\left(\Gamma(\breve{g}_n) - \check{N}(\hat{A}\breve{g}_n, \hat{C}\breve{g}_n, \hat{E}\breve{g}_n, \hat{U}\breve{g}_n)\right)\right]\right\}, \quad (9)$$

Assume that $\{\acute{\alpha}_n\}, \{\acute{\varepsilon}_n\}, \{\ddot{\alpha}_n\}, \{\ddot{\varepsilon}_n\}$ are sequences in [0, 1] holding the next conditions:

- $\acute{\alpha}_n + \acute{\varepsilon}_n = 1, \ddot{\alpha}_n + \ddot{\varepsilon}_n = 1, \; n \geq 0,$ (10)

- $\lim_{n\to\infty} \acute{\alpha}_n = \lim_{n\to\infty} \ddot{\alpha}_n = 0, \; \sum_{n=1}^{\infty} \acute{\varepsilon}_n = \infty.$ (11)

Now, important main results are presented:

Theorem 2.6. Assume that Ξ be real Hilbert space, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping with nonempty closed convex values, Γ, ϴ: Ξ ⟶ Ξ be a mappings. Let Â, Ĉ, Ê, Û: Ξ ⟶ Ξ Lipschitz continuous mappings with constants ǰ, ħ, ḟ, ƕ respectively, and Ň: $\Xi^4 \longrightarrow \Xi$ be a mapping defined by (2). Suppose that:

- Γ, (Ĩ − Γ) and ϴ are Lipschitz continuous with $\tau, \gamma, \check{z} > 0$, respectively,

- $P_{\overline{\mathfrak{D}}}(\breve{g})$ is ṕ-Lipschitz continuous.

If the next conditions satisfy:

$$\left|ƀ - \frac{\tilde{e}đ - á - đ}{\hat{a}^2ǰ^2 - đ^2}\right| < \frac{\sqrt{(4\tilde{e} - á - đ)^2 - (\hat{a}^2ǰ^2 - đ^2)\tilde{e}(2 - \tilde{e})}}{\hat{a}^2ǰ^2 - đ^2}$$

$\tilde{e}đ > á + đ + \sqrt{(\hat{a}ǰ - đ)(\hat{a}ǰ + đ)\acute{e}(2 - \tilde{e})}, \; \tilde{e}đ > á + đ, \; \hat{a}ǰ > đ$

which in $\tilde{e} = 2\gamma + ṕ + \check{z}$, $đ = ρ́ + \hat{u}ƕ$, and $\vartheta = \tilde{e} + \sqrt{1 + 2ƀá + ƀ^2\hat{a}^2ǰ^2} + ƀđ < 1$ (12)

Then there exists a unique $\breve{g} \in \Xi$ holding GSNIQVIP (1). .

Proof: It is enough to show that the mapping in (5) has a unique fixed point in Ξ.

Let $\breve{g}, \breve{y} \in \Xi$, by Lemma (1.4) and Lipschitz continuity of $P_{\overline{\mathfrak{D}}}(\breve{g})$ and (Ĩ − Γ), we yield

$$\|\tilde{O}(\breve{g}) - \tilde{O}(\breve{y})\|$$
$$= \left\| \breve{g} - \Gamma(\breve{g}) + P_{\overline{\mathfrak{D}}}(\breve{g}) \left[\Gamma(\breve{g}) - ƀ \left(\Gamma(\breve{g}) - \check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right] - \breve{y} + \Gamma(\breve{y}) - P_{\overline{\mathfrak{D}}}(\breve{y}) \left[\Gamma(\breve{y}) - ƀ \left(\Gamma(\breve{y}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{y}) \right) \right] \right\|$$

$$\leq \left\| \breve{g} - \Gamma(\breve{g}) - (\breve{y} - \Gamma(\breve{y})) \right\| + ṕ\|\breve{g} - \breve{y}\| + \left\| P_{\overline{\mathfrak{D}}}(\breve{y}) \left[\Gamma(\breve{g}) - ƀ \left(\Gamma(\breve{g}) - \check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right] - P_{\overline{\mathfrak{D}}}(\breve{y}) \left[\Gamma(\breve{y}) - ƀ \left(\Gamma(\breve{y}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{y}) \right) \right] \right\|$$

$$\leq 2\left\| \breve{g} - \Gamma(\breve{g}) - (\breve{y} - \Gamma(\breve{y})) \right\| + ṕ\|\breve{g} - \breve{y}\| + ž\|\breve{g} - \breve{y}\| + \left\| (\breve{g} - \breve{y}) + ƀ \left(\check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| \Gamma(\breve{g}) - \Gamma(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| e(\breve{g}) - e(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{y}) \right\|$$

$$\leq (2\gamma + ṕ + ž)\|\breve{g} - \breve{y}\| + \left\| (\breve{g} - \breve{y}) + ƀ \left(\check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| \Gamma(\breve{g}) - \Gamma(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| e(\breve{g}) - e(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\| + ƀ \left\| \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{y}) \right\| \quad (13)$$

By, $\check{N}$ is á-strongly monotone with respect to $\hat{A}$ in $\check{N}_1$, Lipschitz continuous with respect to $\check{N}_1$, and $\hat{A}$ is Lipschitz continuous mapping, we get

$$\left\| (\breve{g} - \breve{y}) + ƀ \left(\check{N}(\hat{A}\breve{g}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\|$$

$$\leq \|\breve{g} - \breve{y}\|^2 + 2ƀá\|\breve{g} - \breve{y}\|^2 + ƀ^2 â^2 \left\| \hat{A}\breve{g} - \hat{A}\breve{y} \right\|^2$$

$$\leq (1 + 2ƀá + ƀ^2 â^2 ĵ^2)\, \|\breve{g} - \breve{y}\|^2 \quad (14)$$

Since $\check{N}$ is Γ -strongly monotone with respect to $\hat{C}$ in $\check{N}_2$, Lipschitz continuous with respect to $\check{N}_2$, and $\hat{C}$ is Lipschitz continuous, we yield

$$\left\| \Gamma(\breve{g}) - \Gamma(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{g}, \hat{E}\breve{g}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\|^2$$

$$\leq \tau\|\breve{g} - \breve{y}\|^2 - 2ć\|\Gamma(\breve{g}) - \Gamma(\breve{y})\|^2 + ĉ^2 \left\| \hat{C}\breve{g} - \hat{C}\breve{y} \right\|^2$$

$$\leq (\tau^2(1 - 2ć) + ĉ^2 ħ^2)\|\breve{g} - \breve{y}\|^2 \quad (15)$$

$$\left\| e(\breve{g}) - e(\breve{y}) - \left(\check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{y}, \hat{U}\breve{g}) - \check{N}(\hat{A}\breve{y}, \hat{C}\breve{y}, \hat{E}\breve{g}, \hat{U}\breve{g}) \right) \right\|^2$$

$\leq ž^2\|ğ - ÿ\|^2 - 2ẽ\|\mathrm{e}(ğ) - \mathrm{e}(ÿ)\|^2 + ẽ^2\left\|Êğ - Êÿ\right\|^2$

$\leq (ž^2(1 - 2ẽ) + ẽ^2ḟ^2)\|ğ - ÿ\|^2$ (16)

$\|Ň(Âÿ, Ĉÿ, Êÿ, Ûğ) - Ň(Âÿ, Ĉÿ, Êÿ, Ûÿ)\| \leq ûђ\|ğ - ÿ\|$ (17)

From (14), (15), (16), (17)

$\|Õ(ğ) - Õ(ÿ)\| \leq \vartheta\|ğ - ÿ\|, \forall ğ, ÿ \in \Xi$ (18)

$\vartheta = ẽ + \sqrt{1 + 2ƀá + ƀ^2â^2ĵ^2} + ƀđ,$

$ẽ = 2\,\gamma + ṕ + ž, ὀ = \sqrt{\tau^2(1 - 2ć) + ĉ^2ħ^2} + \sqrt{ž^2(1 - 2ẽ) + ẽ^2ḟ^2}, đ = ὀ + ûђ$

By (12) we know $0 < \vartheta < 1$, thus ∃ unique fixed point ğ ∈ Ξ of Õ, s.t ğ is a unique solution of GNIQVIP (3).

Theorem 2.7. Assume that the hypotheses of Theorem (2.6) hold. Then $ğ_n \longrightarrow ğ$, as $n \to \infty$, $\{ğ_n\}$ defined by Algorithm (2.4).

Proof. From (6), (18), and ğ ∈ Õ, we yield

$\|ğ_{n+1} - ğ\| =$

$\left\|\left[ÿ_n - \Gamma(ÿ_n) + P_{\mathfrak{D}}(ÿ_n)\left[\Gamma(ÿ_n) - ƀ\left(\Gamma(ÿ_n) - Ň\big(Âÿ_n, Ĉÿ_n, Êÿ_n, Ûÿ_n\big)\right)\right] + \eta_n ú_n\right] - ğ\right\|$

$$= \left\|\left[ÿ_n - \Gamma(ÿ_n) + P_{\mathfrak{D}}(ÿ_n)\left[\Gamma(ÿ_n) - ƀ\left(\Gamma(ÿ_n) - Ň\big(Âÿ_n, Ĉÿ_n, Êÿ_n, Ûÿ_n\big)\right)\right] + \eta_n ú_n\right] - \left[ğ - \Gamma(ğ) + P_{\mathfrak{D}}(ğ_n)\left[\Gamma(ğ) - ƀ\left(\Gamma(ğ) - Ň\big(Âğ, Ĉğ, Êğ, Ûğ\big)\right)\right] + \eta_n ğ\right]\right\|$$

$\leq \|Õ(ÿ_n) - Õ(ğ)\| + \eta_n\|ú_n - ğ\|$

$\leq \vartheta\|ÿ_n - ğ\| + \eta_n\|ú_n - ğ\|$ (19)

By, the same way, we get

$$\|ÿ_n - ğ\| = \left\|\left[ά_n\left\{ğ_n - \Gamma(ğ_n) + P_{\mathfrak{D}}(ğ_n)\left[\Gamma(ğ_n) - ƀ\left(\Gamma(ğ_n) - Ň\big(Âğ_n, Ĉğ_n, Êğ_n, Ûğ_n\big)\right)\right]\right\} + έ_n\left\{t_n - \Gamma(t_n) + P_{\mathfrak{D}}(t_n)\left[\Gamma(t_n) - ƀ\left(\Gamma(t_n) - Ň\big(Ât_n, Ĉt_n, Êt_n, Ût_n\big)\right)\right]\right\} + ή_n \acute{v}_n\right] - ğ\right\|$$

$$= \left\|ά_n\left\{ğ_n - \Gamma(ğ_n) + P_{\mathfrak{D}}(ğ_n)\left[\Gamma(ğ_n) - ƀ\left(\Gamma(ğ_n) - Ň\big(Âğ_n, Ĉğ_n, Êğ_n, Ûğ_n\big)\right)\right]\right\} - ά_n\left\{ğ - \Gamma(ğ) + P_{\mathfrak{D}}(ğ)\left[\Gamma(ğ) - ƀ\left(\Gamma(ğ) - Ň\big(Âğ, Ĉğ, Êğ, Ûğ\big)\right)\right]\right\} + έ_n\left\{t_n - \Gamma(t_n) + P_{\mathfrak{D}}(t_n)\left[\Gamma(t_n) - ƀ\left(\Gamma(t_n) - Ň\big(Ât_n, Ĉt_n, Êt_n, Ût_n\big)\right)\right]\right\} - έ_n\left\{ğ - \Gamma(ğ) + P_{\mathfrak{D}}(ğ)\left[\Gamma(ğ) - ƀ\left(\Gamma(ğ) - Ň\big(Âğ, Ĉğ, Êğ, Ûğ\big)\right)\right]\right\} + ή_n[\acute{v}_n - ğ]\right\|$$

$\leq ά_n\|Õ(ğ_n) - Õ(ğ)\| + έ_n\|Õ(t_n) - Õ(ğ)\| + ή_n\|\acute{v}_n - ğ\|$

$\leq ά_n\vartheta\|ğ_n - ğ\| + έ_n\vartheta\|t_n - ğ\| + ή_n\|\acute{v}_n - ğ\|$ (20)

Also,

$\|\mathcal{t}_n - \breve{g}\| \leq \H{\alpha}_n\|\breve{g}_n - \breve{g}\| + \H{\varepsilon}_n\vartheta\|\breve{g}_n - \breve{g}\| + \H{\eta}_n\|\acute{w}_n - \breve{g}\|$ (21)

Let $\acute{Y} = max\{sup_n\|\acute{u}_n - \breve{g}\|, sup_n\|\acute{v}_n - \breve{g}\|, sup_n\|\acute{w}_n - \breve{g}\|, n \geq 0\ \}$ (22)

Then, $\acute{Y} < \infty$, and by applying Lemma (1.2) get the next:

$\|\mathcal{t}_n - \breve{g}\| \leq \H{\alpha}_n\|\breve{g}_n - \breve{g}\| + \H{\varepsilon}_n\vartheta\|\breve{g}_n - \breve{g}\| + \H{\eta}_n\acute{Y}$

$\|\breve{y}_n - \breve{g}\| \leq \acute{\alpha}_n\vartheta\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\vartheta\H{\alpha}_n\|\breve{g}_n - \breve{g}\| + \H{\varepsilon}_n\acute{\varepsilon}_n\vartheta^2\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\vartheta\H{\eta}_n\acute{Y} + \acute{\eta}_n\acute{Y}$

$\|\breve{g}_{n+1} - \breve{g}\| \leq \acute{\alpha}_n\vartheta^2\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\vartheta^2\H{\alpha}_n\|\breve{g}_n - \breve{g}\| + \H{\varepsilon}_n\acute{\varepsilon}_n\vartheta^3\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\vartheta^2\H{\eta}_n\acute{Y}$

$+\acute{\eta}_n\vartheta\acute{Y} + \eta_n\acute{Y}$

$\leq [\acute{\alpha}_n\vartheta^2 + \vartheta\acute{\varepsilon}_n(\H{\alpha}_n\vartheta + \H{\varepsilon}_n\vartheta^2)]\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\vartheta^2\H{\eta}_n\acute{Y} + (\acute{\eta}_n\vartheta + \eta_n)\acute{Y}$

$\leq [1 - \acute{\varepsilon}_n + \vartheta\acute{\varepsilon}_n]\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\H{\eta}_n\acute{Y} + (\acute{\eta}_n + \eta_n)\acute{Y}$

$\leq [1 - (1 - \vartheta)\acute{\varepsilon}_n]\|\breve{g}_n - \breve{g}\| + \acute{\varepsilon}_n\H{\eta}_n\acute{Y} + (\acute{\eta}_n + \eta_n)\acute{Y}$

Let $\Delta_n = (1 - \vartheta)\acute{\varepsilon}_n$, $\daleth_n = \|\breve{g}_n - \breve{g}\|$, $\varepsilon_n = \acute{\varepsilon}_n\H{\eta}_n\acute{Y}$, and $\sigma_n = (\acute{\eta}_n + \eta_n)\acute{Y}$.
Then, from conditions of Theorem (2.6), we get $\sum_{n=1}^{\infty}\Delta_n = \infty$, $\sum_{n=1}^{\infty}\sigma_n < \infty$, and $\lim_{n\to\infty}\varepsilon_n = 0$.
Thus, $\breve{g}_n \longrightarrow \breve{g}$, as $n \to \infty$.

Theorem 2.8. Assume that Ξ be real Hilbert space, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping with nonempty closed convex values, Γ, ϴ: Ξ ⟶ Ξ be a mappings. Let Â, Ĉ, Ê, Û: Ξ ⟶ Ξ Lipschitz continuous mappings with positive constants ǰ, ħ, ḟ, ƕ respectively, and Ň: $\Xi^4 \to \Xi$ be a mapping defined by (2). Suppose that:

- Γ, (Ĩ − Γ) and ϴ are Lipschitz continuous with positive constants τ, γ, ž, respectively,

- $P\overline{\mathfrak{D}}(\breve{g})$ is ṕ-Lipschitz continuous.

If the condition (12) in Theorem (2.6) satisfy. Then there exists a unique ğ ∈ Ξ holding problem (3), and $\breve{g}_n \longrightarrow \breve{g}$, as $n \to \infty$, $\{\breve{g}_n\}$ is defined by Algorithm (2.5).

Theorem 2.9. Assume that $\emptyset \neq \overline{\mathfrak{D}} \subset \Xi$, ḿ: Ξ ⟶ Ξ be a Lipschitz continuous mapping with $\varpi > 0$, for $ṕ = 2\varpi$, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping, Γ, ϴ: Ξ ⟶ Ξ be a mappings such that Γ, (Ĩ − Γ) and ϴ are Lipschitz continuous with positive constants τ, γ, ž, respectively. Let Â, Ĉ, Ê, Û: Ξ ⟶ Ξ Lipschitz continuous mappings with positive constants ǰ, ħ, ḟ, ƕ respectively, and Ň: $\Xi^4 \to \Xi$ be a mapping define by (2). If the conditions in (12) of Theorem (2.6) satisfy, then there exists a unique ğ ∈ Ξ holding problem (4), and $\breve{g}_n \longrightarrow \breve{g}$, as $n \to \infty$, $\{\breve{g}_n\}$ defined by Algorithm (2.4).

Theorem 2.10: Assume that $\emptyset \neq \overline{\mathfrak{D}} \subset \Xi$, ḿ: Ξ ⟶ Ξ be a Lipschitz continuous mapping with $\varpi > 0$, for $ṕ = 2\varpi$, $\overline{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}$ is a set-valued mapping, Γ, ϴ: Ξ ⟶ Ξ be a mappings such that Γ, (Ĩ − Γ) and ϴ are Lipschitz continuous with positive constants τ, γ, ž, respectively. Let Â, Ĉ, Ê, Û: Ξ ⟶ Ξ Lipschitz continuous mappings with positive constants ǰ, ħ, ḟ, ƕ respectively, and Ň: $\Xi^4 \to \Xi$ be a mapping define by (2). If the conditions in (12) of Theorem (2.6) satisfy, then there exists a unique ğ ∈ Ξ holding problem (4), and $\breve{g}_n \longrightarrow \breve{g}$, as $n \to \infty$, $\{\breve{g}_n\}$ defined by Algorithm (2.5).

- The proof of Theorems (2.8), (2.9) and (2.10), by the same way of Theorems (2.6) and (2.7).

Conclusion

The effect of this work betokens that the new iterative schemes holding some appropriate conditions has a unique solution by a technique of fixed point. Also, from these iterative schemes gives several results when it be with errors and without errors of Lipschitz continuous mappings and strongly monotone mapping for solving GSNIQVIP and has results of these iterative schemes for solving special case of GSNIQVI with errors and without errors. The solution of iterative schemes via the technique of fixed point by showing the stipulations referred to above warranting the convergence of the manner.

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الخوارزميات التكرارية لفئة من المتراجحات شبه التغاير الضمنية

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معلومات المؤلف

الايميل:
الموبايل:

الخلاصة:

من المعروف أن إحدى المسائل الأكثر أهمية وتعقيدًا في نظرية متراجحة التغاير (VIT) هي تطوير مخططات تقريبية فعالة وقابلة للتنفيذ لحل فئات متنوعة من متراجحة التغاير. وبالتالي، من خلال هذه الدراسة، نقدم مخططات تكرارية جديدة بها أخطاء لحل متراجحات شبه التغاير الضمنية غير الخطية المعممة بقوة (GSNIQVI) بواسطة نقطة ثابتة معرفة لتطبيق مجموعة القيمة $\bar{\mathfrak{D}} \subset \Xi, \bar{\mathfrak{D}} : \Xi \longrightarrow 2^{\Xi}\emptyset \neq$ مع قيم محدبة مغلقة في فضاء هلبرت Ξ. تم تصميم هذه المخططات التكرارية لتطبيقات لبشز المستمرة، Γ - الرتيبة بقوة، والتطبيقات e-الرتيبة بقوة في ظل بعض الظروف المناسبة. تعمل هذه المخططات التكرارية على حل حالة خاصة من GSNIQVI مع وجود أخطاء وبدون أخطاء. لقد أثبتنا وجود نتائج وحيدة لحل GNIQVIP وتقارب الخوارزميات التكرارية الجديدة مع وجود أخطاء، وبدون أخطاء، لعدة تطبيقات: الأول هو $\Gamma: \Xi \longrightarrow \Xi$ لتطبيق لبشز المستمر، والثاني $\hat{A}, \hat{C}, \hat{E}, \hat{U}: \Xi \longrightarrow \Xi$ عبارة عن تطبيقات لبشز المستمرة، والثالث $\check{N}: \Xi^4 \longrightarrow \Xi$ هو تطبيق بحيث يكون رتيب بقوة فيما يتعلق بـ $\hat{A}$ في $\check{N}_1$ ، Γ- رتيب بقوة فيما يتعلق بـ $\hat{C}$ في $\check{N}_2$، e - رتيب بقوة فيما يتعلق بـ $\hat{E}$ في $\check{N}_{31}$ ومستمر لبشز فيما يتعلق بـ $\check{N}_1$، $\check{N}_2$، $\check{N}_3$، $\check{N}_4$ والرابع $P_{\bar{\mathfrak{D}}}(\breve{g}) \; \Xi \rightarrow \bar{\mathfrak{D}}:$ هو الإسقاط المتري وهو مستمر لبشز. تم إستيحاء نتائجنا وتشجيعها للعديد من الأعمال البحثية في المصادر.