

## On Soft Set Representations of Simple Graphs

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### Abstract

Within this article, we investigate a method of representing simple graphs using soft set theory. This method is proposed in [4], which uses soft sets to represent simple graphs depending on neighbourhood relations of vertices. We explore the representation of various simple graph types using soft sets and present several new findings. We also introduce the soft set representative vector and present some interesting results.

## 1. Introduction

Representing graphs using specific objects is a vital subject in the theory of graphs. The classical representations of graphs depend on using two simple objects: small circles or discs and simple curves. Circles or discs embody the vertices and curves embody the edges of the graph under consideration. The way of linking circles between each other using curves gives all the needed information to describe such a graph. However, the extensions of using graph theory tools in many other disciplines still motivate researchers to introduce other types of representations of graphs using different objects. The adjacent matrix and the incident matrix are two other classical methods of representing graphs. Such two representations of graphs are powerful tools in hand of dealing with graphs in various aspects of computer science, just to name one benefit of them. However, many other representations of graphs have been proposed using, for example, geometric objects or sets. The geometric objects aligned with each other according to the neighbouring of the vertices they represent. A typical example following this model of representing graphs is the result of Keobe-Andreev-Thurston, [1] and [2]. This work, however, concerns representing simple graphs using a specific kind of set, namely soft set.

Representing graphs using specific objects involves studying structural properties and algorithmic problems according to the class of graphs under consideration. The common method of studying representing a class of graphs usually required finding a combinatorial inductive construction from which the whole class of graphs can be built. Then, one can find a way of representing the elements of such an inductive construction using certain objects. See for example

[3]. Since graph theory facilitates describing the relationships between sets and their elements. Such a virtue naturally leads to find possible ways to represent graphs using sets. This work concerns with investigating further results of representing graphs using soft sets. Soft set theory emerged to tackle problems involve uncertainty data. In [4], such theory appeared as a new promising tool to resolve difficulties faced other related theories such as fuzzy sets [5] and rough sets [6]. Soft set theory in comparison with other similar theories, does not required pre-processing of the data under consideration, [7]. Soft graphs received a lot of consideration and one can consult [8] for further reading. Representing graphs using soft sets introduced in [9] and this work explores additional properties of soft set representation of graphs and presents some interesting results. The rest sections of this article include the following: Section 2 includes basic but fundamental concepts in graph theory and soft set theory. Section 3 deals with representing graphs using soft sets with some interesting properties of such representation. In section 4, additional results regarding soft set representations of graphs were explored.

## 2. Preliminaries

This section is designed to present some fundamental concepts in graph theory, and provide a short review that covers some concepts of soft sets.

### 2.1 Graph theory background

A graph  $G = (V, E)$  consists of a finite set of vertices  $V$  and a set of edges  $E$  where edges are unordered pairs of two vertices [10].

**Assumption:** In this article, by  $G = (V, E)$  is meant a simple graph and both  $V$  and  $E$  are finite.

An edge  $e_{uv}$  is unordered pair  $\{u, v\}$  which is incident to both  $u$  and  $v$ . Consequently, if  $u, v \in V$  are adjacent, then  $e_{uv} = e_{vu}$ . This article concerns only with simple graphs that have neither multi-edges nor loops. The *degree* of  $v$  in  $V$ ,  $deg(v)$ , is the cardinality of the set  $\{e_{uv} \in E : u \in V\}$ , that is,  $deg(v) = |\{e_{uv} \in E : u \in V\}|$ .

The following concepts can be found in [11]. An *r-regular* graph  $G = (V, E)$  is a graph in which  $deg(v) = r$  for each  $v \in V$ . The *complete* graph  $K_n$  is a graph  $G = (V, E)$  such that for every two vertices  $u, v \in V$  there is exactly an edge  $e_{uv}$  joining them and  $|V| = n$ .  $G = (V, E)$  is a *bipartite* providing that  $V = M \cup N$ ,  $M \cap N = \emptyset$  and  $E = \{e_{uw} \in E : u \in M \text{ and } w \in N\}$ . A bipartite graph  $G$  is called complete if for every  $u \in M$  and every  $w \in N$ , there is an edge  $e_{uw}$  joins  $u$  and  $w$ .

**Theorem 2.1.** [12] In any graph  $G = (V, E)$ , the cardinality of  $E$  is equals to  $(\sum_{v \in V} deg(v))/2$ .

### 2.2. Soft set theory background

We recall some fundamental concepts regarding soft sets.

**Definition 2.2.** [10] Let  $E$  be a set of parameters and  $A \subset E$ . A *soft set*,  $(F, A)$ , on the universe set  $X$  consists of the set of ordered pairs  $(F, A) = \{(e, F(e)) : e \in E, F(e) \in P(X)\}$  where the function  $F : E \rightarrow P(X)$  is defined by  $F(e) = \emptyset$  if  $e \notin A$  where  $P(X)$  is the power set of  $X$ .

The above function  $F$  is said to be an approximate function of  $(F, A)$ .

**Example 2.3.** Consider the universe  $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$  and let  $E = \{e_1, e_2, e_3, e_4, e_5\}$  be the decision parameters sets. Now, let  $A = \{e_2, e_3, e_5\}$  and  $F(e_2) = \{x_2, x_6\}$ ,  $F(e_3) = \{x_1, x_3, x_4\}$  and  $F(e_5) = \{x_1, x_7, x_8\}$ .

Therefore, our soft set is  $(F, A) = \{(e_2, \{x_2, x_6\}), (e_3, \{x_1, x_3, x_4\}), (e_5, \{x_1, x_7, x_8\})\}$ .

The following concepts can be found in [13]. If  $F(e) = \emptyset \forall e \in A$ , then such a soft set with this approximate function is called an *empty* soft set. An  $A$ -universe set is a soft set such that  $F(e) =$

$X$  for all  $e \in A$ . In case that  $A = E$ , then  $(F, A)$  is called a *universe* soft set. Now, suppose that  $(F, A)$  and  $(G, B)$  are soft sets.  $(G, B)$  is a soft subset of  $(F, A)$  whenever  $G(e) \subseteq F(e)$  for all  $e \in B$ . Such an operation is denoted by  $(G, B) \subseteq (F, A)$ . If it happens that  $(G, B) \neq (F, A)$ , then  $(G, B)$  is a soft proper subset of  $(F, A)$ . We denote such implication as  $(G, B) \subsetneq (F, A)$ . Finally, let  $(F, A)$  be a soft set on the universe set  $X$ . The soft complement of  $(F, A)$ , is defined by  $F^{\tilde{c}}(e) = X - F(e)$  for all  $e \in A$ . Such an operation is denoted by  $(F, A)^{\tilde{c}}$ .

### 3. Soft set representations of graphs

This section examines some properties of soft set representation of graphs. Moreover, some new concepts and notations will be introduced.

**Definition 3.1.** [4] Given the graph  $G = (V, E)$ . Define  $F_G : V \rightarrow P(V)$  such that

$$F_G(v) = \begin{cases} \{u \in V, \text{ if } e_{uv} \in E \\ \emptyset, \text{ if } e_{uv} \notin E \end{cases}$$

for every  $v \in V$ . The function  $F_G$  defines a soft set  $(F_G, V)$  over the power set  $P(V)$ .  $(F_G, V)$  is called the *soft set representation* of  $G$ , shortly *SSR* of  $G$ .

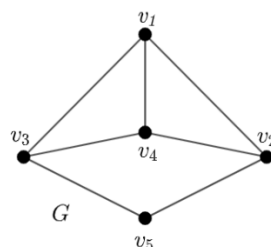
The image of  $v \in V$ ,  $F_G(v)$ , is called  $F_G$ -image of  $v$ .  $F_G$ -image of an isolated vertex  $v$  in  $G$  whose soft set representation  $(F_G, V)$  is the empty set, i.e.  $F_G(v) = \emptyset$ . Observe that  $x \in F_G(y)$  if and only if  $y \in F_G(x)$ . We denote the collection of all  $F_G$ -images of an *SSR* of  $G$  by  $\mathcal{F}_G$ . Notice that  $|\mathcal{F}_G| = |V|$ . Now, if  $V$  is partitioned into two sets  $M$  and  $N$ . Then we denote  $\mathcal{F}_{G,M}$  and  $\mathcal{F}_{G,N}$  the collection of  $F_G$ -images of all  $u \in M$  and  $w \in N$ , respectively. In the following example, we illustrate some of the concepts declared above.

Let  $e(F_G)$  and  $o(F_G)$  be respectively symbols to denote how many  $|F_G(V)|$  are odd or even.

**Example 3.2.** Consider the *SSR* of  $G$  were

$(F_G, V) = \{(v_1, \{v_2, v_3, v_4\}), (v_2, \{v_1, v_4, v_5\}), (v_3, \{v_1, v_4, v_5\}), (v_4, \{v_1, v_2, v_3\}), (v_5, \{v_2, v_3\})\}$ . Then,  $|F_G(v_1)| = |F_G(v_4)| = |F_G(v_2)| = |F_G(v_3)| = 3$  and  $|F_G(v_5)| = 2$ . Moreover,  $e(F_G) = 1$ ,  $o(F_G) = 4$ . The retrieving of  $G$  is depicted in Figure 1.

Let  $(F_G, V)$  be the *SSR* of  $G$  and  $v \in V$ .  $e_v$  refers to an edge that is incident to  $v$ . The edge-incident set of all edges to the vertex  $v$  is defined as  $E_{F_G(v)} = \{e_{vx} \in E : x \in F_G(v)\}$ . Clearly,  $|E_{F_G(v)}| = |F_G(v)|$ . Let  $\mathcal{E}_G = \{E_{F_G(v)} : v \in V\}$  be the collection of all edge-incident sets of  $G$ . One can easily observe that  $|\mathcal{E}_G| = |V|$ . If  $M \subset V$ , then  $\mathcal{E}_{G,M}$  refers to the set  $\{E_{F_G(v)} : v \in M\}$ .



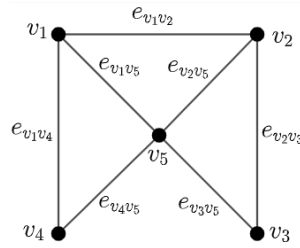
**Fig. 1** A simple graph  $G = (V, E)$ . The soft set representation of this graph is given in Example 3.2.

**Example 3.3.** Let  $(F_G, V)$  be the *SSR* of  $G$  where  $(F_G, V) = \{(v_1, \{v_2, v_4, v_5\}), (v_2, \{v_1, v_3, v_5\}), (v_3, \{v_2, v_5\}), (v_4, \{v_1, v_5\}), (v_5, \{v_1, v_2, v_3, v_4\})\}$ .

So,  $\mathcal{E}_G = \{E_{F_G(v_1)}, E_{F_G(v_2)}, E_{F_G(v_3)}, E_{F_G(v_4)}, E_{F_G(v_5)}\}$  where

$E_{F_G(v_1)} = \{e_{v_1v_2}, e_{v_1v_4}, e_{v_1v_5}\}$ ,  $E_{F_G(v_2)} = \{e_{v_1v_2}, e_{v_2v_3}, e_{v_2v_5}\}$ ,  $E_{F_G(v_3)} = \{e_{v_2v_3}, e_{v_3v_5}\}$ ,

$E_{F_G(v_4)} = \{e_{v_1v_4}, e_{v_4v_5}\}$  and  $E_{F_G(v_5)} = \{e_{v_1v_5}, e_{v_2v_5}, e_{v_3v_5}, e_{v_4v_5}\}$



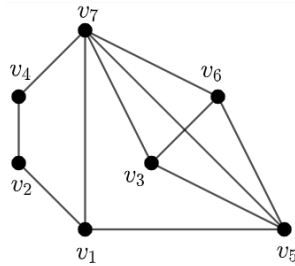
**Fig. 2** A simple graph  $G = (V, E)$ . The soft set representation of this graph is given in Example 3.3.

Let  $(F_G, V)$  be the SSR of a graph  $G = (V, E)$ . An  $F_G$ -image vector  $\mathbf{f}_G = [f_1 f_2 \dots f_{|V|}]$  of  $(F_G, V)$  is a sequence of the cardinalities of  $F_G$ -images of  $(F_G, V)$ . Every component of  $\mathbf{f}_G$  represents the cardinality of an  $F_G$ -image of a vertex in  $G$ .

**Example 3.4.** Let  $(F_G, V)$  be the SSR of the graph  $G$  which is depicted in Figure 3, where

$$(F_G, V) = \{(v_1, \{v_2, v_5, v_7\}), (v_2, \{v_1, v_4\}), (v_3, \{v_5, v_6, v_7\}), (v_4, \{v_2, v_7\}), \\ (v_5, \{v_1, v_3, v_6, v_7\}), (v_6, \{v_3, v_5, v_7\}), (v_7, \{v_1, v_3, v_4, v_5, v_6\})\}.$$

Then,  $\mathbf{f}_G = [3 \ 2 \ 3 \ 2 \ 4 \ 3 \ 5]$ .



**Fig. 3** A simple graph  $G = (V, E)$ . The soft set representation of this graph is given in Example 3.4.

Let  $\mathbf{v}$  be a vector, then  $\mathbf{v}$  is called a *soft set representative* if there is an SSR of  $G$ , say  $(F_G, V)$  so that  $\mathbf{v}$  is the  $F_G$ -image vector of  $(F_G, V)$ , i.e.  $\mathbf{v} = \mathbf{f}_G$ .

It worth to mention that the three examples, 3.2, 3.3 and 3.4 present the SSR of each corresponding given graph. However, Example 3.3 clarifies also how the collection of all edge-incident sets of  $G$  can be depicted while Example 3.4 depicts the  $F_G$ -image vector of the given graph.

In the following, we present properties of some soft set representations of some special graphs

**Complete graphs:** Consider the complete graph  $K_n = G = (V, E)$  where  $n$  is positive integer number. The set  $(F_G, V) = \{(v_i, V - \{v_i\}) : i = 1, 2, \dots, |V| = n\}$  is the SSR of  $G$ .

- a)  $|F_G(v)| = |V| - 1$  for every  $v \in V$ .
- b)  $|E_{F_G(v)}| = |V| - 1$  for every  $v \in V$ .
- c)  $\mathbf{f}_G = [n - 1 \ n - 1 \ \dots \ n\text{-times}]$ .

**Regular graphs:** Suppose that  $(F_G, V)$  be the SSR of  $G$  where  $G$  is an  $r$ -regular with  $r$  is a positive integer number and  $r \leq |V|$ . Then

- a)  $|F_G(v)| = r$  for every  $v \in V$ .
- b)  $|E_{F_G(v)}| = r$  for every  $v \in V$ .
- c)  $\mathbf{f}_G = [r \ r \ \dots \ |V|\text{-times}]$ .

**Complete bipartite graphs:** Consider the bipartite graph  $G = (V, E) = K_{k,l}$  with  $V = M \cup N$ ,  $M \cap N = \emptyset$ ,  $|M| = k$  and  $|N| = l$ . Let  $(F_G, V)$  be the SSR of  $G$ .

Then  $|E_{F_G(v)}| = k$  for every  $v \in M$  and  $|E_{F_G(v)}| = l$  for every  $v \in N$ .

**Wheel graphs:** Let  $(F_G, V)$  be the SSR of  $G$  where  $G$  is the wheel graph  $W_n$ ,  $n = |V|$  and  $c$  be the center vertex of  $G$ . Then

- a)  $|F_G(c)| = n - 1$  and  $|F_G(v)| = 3$  for every  $v \in V - \{c\}$ .
- b)  $\mathbf{f}_G = [n - 1 \ 3 \ \dots \ 3]$ .

**Star graphs:** Let  $G = S_n$  be the star graph where  $n$  is a positive integer number and  $v_1$  be the center of  $G$ . Let  $(F_G, V)$  be the SSR of  $G$ . Then

- a)  $|F_G(v_1)| = n - 1$  and  $|F_G(v)| = 1$  for every  $v \in V - \{v_1\}$ .
- b)  $|E_{F_G(v_1)}| = n - 1$  and  $|E_{F_G(v)}| = 1$  for every  $v \in V - \{v_1\}$ .
- c)  $\mathbf{f}_G = [n - 1 \ 1 \ 1 \ \dots \ 1]$  where the length of  $\mathbf{f}_G$  is equal to  $n$ .

**Cycle graphs:** Consider the cycle graph  $G = C_n$  where  $n$  is a positive integer number. Let  $(F_G, V)$  be the SSR of  $G$ . Then

- a)  $|F_G(c)| = 2$  for every  $v \in V$ , see Lemma 4.2.
- b)  $\mathbf{f}_G = [2 \ 2 \ \dots \ 2]$  where the length of  $\mathbf{f}_G$  is equal to  $n$ .

**Null graphs:** Let  $(F_G, V)$  be the SSR of the empty graph  $G = (V, E)$  with  $E = \emptyset$ , then  $\mathcal{E}_G = \emptyset$ .

**Lemma 3.5.** Let  $(F_G, V)$  be the SSR of  $G$  and  $v \in V$ . Then  $\deg(v) = |F_G(v)|$ .

Proof. The definition of  $F_G$ -image set gives the proof.

**Proposition 3.6.** [4] Let  $(F_G, V)$  be the SSR of  $G$ . Then sum of the degrees of  $G$  is equal to  $\sum_{v \in V} |F_G(v)|$ .

**Lemma 3.7.** Let  $(F_G, V)$  be the SSR of  $G$  where  $G$  has order  $n$ . Then,  $|F_G(v)| \leq n - 1$  for every  $v \in V$ .

Proof. Since  $G$  has no loops or multi-edges. So the largest possible degree of each vertex can be  $|V| - 1$ .

**Lemma 3.8.** Let  $(F_G, V)$  be the SSR of a bipartite graph  $G = (V, E)$  where  $V = M \cup N$ ,  $M$  and  $N$  are disjoint. Then, for every two distinct vertices  $u, v \in M$ , we have  $F_G(v) \cap F_G(u) = \emptyset$  and  $E_{F_G(u)} \cap E_{F_G(v)} = \emptyset$ .

**Lemma 3.9.** Let  $(F_G, V)$  be the SSR of an  $r$ -regular graph  $G$ . If  $r$  is odd then  $|\mathcal{F}_G|$  is even.

Proof. Let  $|V| = n$ . So,  $|\mathcal{F}_G| = n$ . Hence,  $rn$  is twice the number of edges. Consequently, if  $r$  is odd then  $n$  is even.

**Lemma 3.10.** Let  $(F_G, V)$  be the SSR of  $G$  where  $G$  is a regular bipartite graph  $G$  with  $V = M \cup N$  and  $M \cap N = \emptyset$ . Then,  $|\mathcal{F}_{G,M}| = |\mathcal{F}_{G,N}|$ .

Proof. Assume that  $|\mathcal{F}_{G,M}| = k$  and  $|\mathcal{F}_{G,N}| = l$  and  $G$  is regular, say  $r$ -regular. Now, consider the vertex set  $M$ . The regularity of  $G$  implies  $|F_G(v)| = r$  for every  $v \in M$ . Then,  $|E| = rk$ . Consider  $N \subset V$ . A similar argument can be proceeded again to get  $|E| = rl$ . Therefore  $rk = rl$  which leads to  $k = l$ .

**Proposition 3.11.** Let  $(F_G, V)$  be the SSR of a complete bipartite graph  $G = (V, E)$  with  $V = M \cup N$  and  $M \cap N = \emptyset$ . Then,  $|E| = |\mathcal{E}_{G,M}| |\mathcal{E}_{G,N}|$ .

Proof. Since  $G$  is bipartite, so  $E_{F_G(u)} \cap E_{F_G(v)} = \emptyset$  for each  $u, v \in M$ . Since  $G$  is complete bipartite graph, so  $|\mathcal{E}_{G,M}| = |M|$ . Similarly,  $|\mathcal{E}_{G,N}| = |N|$ . But  $|E| = |M||N|$ . Therefore,  $|E| = |\mathcal{E}_{G,M}| |\mathcal{E}_{G,N}|$ .

**Proposition 3.12.** The maximum possible value of  $|\mathcal{E}_G|$  in  $(F_G, V)$  of a bipartite graph  $G = (V, E)$  occurs when  $|\mathcal{E}_{G,M}| = |\mathcal{E}_{G,N}|$ .

#### 4. Further Results

In the following, we present more interested results regarding the soft representations of soft sets.

**Proposition 4.1.** Let  $(F_G, V)$  be the SSR of  $G$ . Then,  $o(F_G)$  is even.

Proof. Assume that  $V = R \cup T$  where  $R$  represents the odd vertices set of  $G$  and  $T$  is the even vertices set of  $G$ . Let  $r = \sum_{v \in R} |F_G(v)|$  and  $t = \sum_{v \in T} |F_G(v)|$ .

But  $t$  is even and  $r + t = \sum_{v \in V} |F_G(v)|$  which leads to  $r + t$  is even. Hence,  $r$  should be even. Now, suppose that  $o(F_G) = z$ , so the even number  $r$  is the sum of  $z$  odd numbers. Consequently, we get that  $z$  is even.

**Lemma 4.2.** Let  $(F_G, V)$  be the SSR of a cycle graph  $G$ . Then  $|F_G(v)| = |E_{F_G(v)}| = 2$  for every vertex  $v \in V$ .

Proof. Since  $G$  is cycle graph and  $G$  is simple, so every  $v \in V$  is adjacent to exactly two distinct vertices.

Thus,  $|\mathbf{F}_G(\mathbf{v})| = |\mathbf{E}_{F_G(\mathbf{v})}| = 2$ .

**Lemma 4.3.** Let  $(F_G, V)$  be the SSR of  $G$ . Then the sum of the  $F$ -image vector of  $(F_G, V)$  is even.

Proof. Notice that each component of  $\mathbf{f}_G$  represents a  $F_G$ -image of a vertex in  $G$ . Now, by Proposition 3.6 and Theorem 2.1, we get the required.

**Proposition 4.4.** Let  $(F_G, V)$  be the SSR of  $G$ . Then there exist at least two vertices  $\mathbf{u}, \mathbf{v} \in V$  with  $\mathbf{u} \neq \mathbf{v}$  such that  $|\mathbf{F}_G(\mathbf{u})| = |\mathbf{F}_G(\mathbf{v})|$ .

Proof. Suppose that  $|V| = n$ . So the soft set representative vector  $\mathbf{f}_G$  has exactly  $n$  components. Assume that no two numbers are repeated. Lemma 3.7 asserts that the maximum cardinality of  $F_G$ -image sets can not exceed  $n - 1$ . Hence the soft set representative vector of  $G$  has the form  $\mathbf{f}_G = [n - 1 \ n - 2 \ \dots \ 2 \ 1 \ 0]$ . Thus, if we subtract  $\mathbf{1}$  from the next  $(n - 1)$  components we get a vector has a negative component. This means that  $\mathbf{f}_G$  is not soft set representative vector.

**Proposition 4.5.** Let  $(F_G, V)$  be SSR of  $G$ . Then,  $(F_G^c, V)$  is the SSR of  $G^c$  where  $F_G^c(\mathbf{v}) = V - \{F_G(\mathbf{v}) \cup \{\mathbf{v}\}\}$ .

Proof. Definition of soft complement of a soft set implies the proof.

**Corollary 4.6.** Let  $(F_G, V)$  be the SSR of a complete graph  $G = K_n$  where  $n$  is a positive number. If  $(F_G^c, V)$  is the SSR of  $G^c$ , then  $F_G^c(\mathbf{v}) = \emptyset$  for every  $\mathbf{v} \in V$ .

Proof. Let  $\mathbf{v} \in V$ . By Proposition 4.5,  $F_G^c(\mathbf{v}) = F^c(\mathbf{v}) = V - V = \emptyset$ .

**Corollary 4.7.** Let  $(F_G, V)$  be the SSR of  $G$  where  $G$  is a complete bipartite graph  $K_{m,n} = G = (V, E)$  where  $n$  is a positive number,  $V = M \cup N$  and  $M \cap N = \emptyset$ . If  $(F_G^c, V)$  is SSR of  $G^c$ , then  $F_G^c(\mathbf{u}) = M - \{\mathbf{u}\}$  for every  $\mathbf{u} \in M$  and  $F_G^c(\mathbf{w}) = N - \{\mathbf{w}\}$  for every  $\mathbf{w} \in N$ .

Proof. Since the complement of  $K_{m,n}$  has two components. One component is the complete graph which has the vertex set  $M$  and the other component is the complete graph which has the vertex set  $N$ . Therefore, by Proposition 4.5, if  $\mathbf{u} \in M$  then  $F_{G^c}(\mathbf{u}) = F_G^c(\mathbf{u}) = M - \{\mathbf{u}\}$ . Moreover, if  $\mathbf{w} \in N$  then  $F_{G^c}(\mathbf{w}) = F_G^c(\mathbf{w}) = N - \{\mathbf{w}\}$ .

**Lemma 4.8.** Let  $(F_G, V)$  be the SSR of  $G$ . Then,  $|E| = (\sum_{\mathbf{v} \in V} |\mathbf{E}_{F_G(\mathbf{v})}|)/2$

Proof. We merging Proposition 3.6. and Theorem 2.1 Then we use the fact that  $|\mathbf{E}_{F_G(\mathbf{v})}| = |\mathbf{F}_G(\mathbf{v})|$ .

**Proposition 4.9.** Let  $(F_G, V)$  be the SSR of  $G$ . Suppose that every edge  $e_{uv} \in E$  yields that  $|\mathbf{F}_G(\mathbf{u})|$  is odd and  $|\mathbf{F}_G(\mathbf{v})|$  is even. Then  $G$  is a bipartite graph.

Proof. Let  $M = \{\mathbf{v} \in V : |\mathbf{F}_G(\mathbf{v})| \text{ is odd}\}$  and  $N = \{\mathbf{v} \in V : |\mathbf{F}_G(\mathbf{v})| \text{ is even}\}$ . Clearly  $V = M \cup N$  and  $M \cap N = \emptyset$ . Moreover, every  $e_{xy} \in E$  yields that  $x \in M$  and  $y \in N$ . Therefore,  $G$  is a bipartite graph.

## 5. CONCLUSION

This article studied some unexplored properties of the soft set representations of simple graphs.

Depending on the images of the vertices,  $F_G$ -images, of simple graphs under the approximate functions of the SSR of the graphs, we examined various related results. Besides, we introduced some useful concepts. One of the introduced concepts is the soft set representative. It is a vector of integer numbers where each integer represents the  $F_G$ -image of each vertex of the simple graph under consideration.

## References

1. A. Bondy, U.S.R. Murty, Graph Theory, Graduate Texts in Mathematics, Springer London, (2011).
2. D. Molodtsov, Soft set theory-first results, Comput. Math. Appl., 37, 19-31 (1999). [http://doi.org/10.1016/S0898-1221\(99\)00056-5](http://doi.org/10.1016/S0898-1221(99)00056-5)
3. E. M. Andreev, Convex polyhedra in Lobachevski spaces, Mat. Sb., New Series, 81 (123): 445-478 (1970)
4. J. Cruickshank, D. Kitson, S. Power, Q. Shakir, Topological inductive constructions for tight surface graphs, Graphs and Combinatorics, Graphs and Combinatorics 38, 169, 1-31, (2022).
5. J. D. Thenge, B. S. Reddy and R. S. Jain, Contribution to soft graph and soft tree, New Mathematics and Natural Computation 15(1), 129-143 (2019). DOI: 10.1142/S179300571950008X.
6. J.C.R. Alcantud, A.Z. Khameneh, G. Santos-Garcia, et al. A systematic literature review of soft set theory. Neural Comput and Applic (2024). <https://doi.org/10.1007/s00521-024-09552-x>
7. J.D. Thenge, B.S. Reddy and R.S. Jain, Adjacency and incidence matrix of a soft graph, Communications in Mathematics and Applications 11(1),(2020), 23-30, DOI: 10.26713/cma.v11i1.1281.
8. LA. Zadeh, Fuzzy sets. Information and Control. 8(3): 338-353 (1965). DOI: [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
9. M. Akram and S. Nawaz, "Certain types of soft graphs", U. P. B. Sci., Series A 78(3), 67-82(2016).
10. M. I. Ali, M. Shabir, F. Feng, Representation of graphs based on neighborhoods and soft sets, Int. J. Mach. Learn. Cyber., , 1525-1535 (2017). <http://doi.org/10.1007/s13042-016-0525-z>
11. R. Diestel. Graph theory, volume 173 of Graduate Texts in Mathematics. Springer, Berlin, fifth edition, (2018).
12. W. P. Thurston, The finite Riemann mapping theorem, Invited address, International Symposium in celebration of the proof of the Bieberbach conjecture, Purdue University. (1985)
13. Z. Pawlak Rough sets. International Journal of Computer and Information Sciences. 11: 341-356 (1982). DOI: <https://doi.org/10.1007/BF01001956>.

## حول تمثيلات المجموعات الناعمة للمخططات البيانية البسيطة

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### الخلاصة:

في هذه البحث، نبحث في طريقة تمثيل المخططات البيانية البسيطة باستخدام نظرية المجموعات الناعمة. تم اقتراح هذه الطريقة في [4]، في هذه الطريقة تستخدم المجموعات الناعمة لتمثيل المخططات البيانية البسيطة اعتماداً على علاقات الجوار بين رؤوس المخططات البيانية. نستكشف تمثيل أنواع مختلفة من المخططات البيانية البسيطة باستخدام المجموعات الناعمة ونقدم عدة نتائج جديدة. كما نقدم متجه تمثيلي للمجموعات الناعمة ونعرض بعض النتائج المهمة.

### معلومات البحث:

تاريخ الاستلام:

تاريخ التعديل:

تاريخ القبول:

تاريخ النشر:

### الكلمات المفتاحية:

مخطط بياني بسيط، مجموعة ناعمة،

تمثيل مجموعة ناعمة

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