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Some Topological Indices of the prime graph of commutative ring z_p

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Abstract

In this study, a new concept graph definition known A prime graph of a ring *z_p* denote PG(zp) is present, where the graph's vertices stand in for R's elements s.t, any two vertices a and b adjacent by an edge if and only if and a.b= 0. In this paper, we investigated some new property of PG (z_p) are studied.

Introduction

Graph theory has become a very well-known and rapidly growing field of mathematics due to its extensive theoretical advances and diverse applications to real-world problems. Although graph theory is still a relatively new field of research, it has yielded many profound and novel discoveries over the past 20 years. In biological, social, physical, and information systems, diagrams can be used to represent a variety of relationships and processes.

Sylvester introduced the term graph in an 1878 article in Nature Publishing [1]. The 1st book in graph theory was published by Denes in 1936 [2]. In the last decade of the 19th century, algebraic graph theory began to develop rapidly, and a large number of research articles were published in this branch of graph theory. The fascinating field of algebraic graph theory studies how algebra and graph theory interact. Algebraic methods can be used to prove graph theory facts in surprising and elegant ways. There are many interesting algebraic objects related to graphs. In recent years, the study of algebraic graph theory has become increasingly important. Algebraic graph theory involves converting properties of graphs into algebraic properties and then using algebraic results and techniques to derive theorems about graphs.

However, many algebraic topics can also be understood by converting them into graphs and exploiting the properties of graphs. In (1999), Anderson and Livingston [3] assigned a simple graph to each commutative ring R and examined to interaction between the ring-theoretical conditions and the graph-theoretic properties of . Akbari et al.. In (2010) Bhavanari et al. [3], they considered binding rings R (not necessarily commutative) and defined a new concept “primary graph of R” (denoted by PG(R)). They presented some examples and obtained some basic important results related to PG(R). (2013) Chelvam and Asir. [5] studied the advantages in commutative ring ensemble graphs. In (2014), Patra and Kalita [6] studied prime graphs of commutative rings. In (2023), Nermen J. Khalel andNabeel E. Arif [7] studied Associate graph of a commutative ring

In the articles, we study more topological metrics that need degrees of examples: eccentricity index [8], connectivity index [9], sum connectivity index [10], 1st and 2nd index [11], forgetting indicator [12], exponential geometric arithmetic [13], atomic bond connectivity index [13] and harmonic index [13]. Topological indexing of symmetry group graphs was introduced in (2019) Abdussakir [14], also examined in (2020) G. R. Roshini [15], and also in (2020) G. R. Roshini [16] , Alaa. J and Akram .S [17] in (2021) study topological indices and (Schultz and Hosoya) polynomials of the intersection graph of subgroup of the group Zr.

Results:

Definition 2.1: Let R a ring. A prime graph of ring R indicated as PG($\boldsymbol{R}$) is a graph where V(PG($\boldsymbol{R}$))= R and E(PG($\boldsymbol{R}$))=$\{\overline{ab}\ ,\ a.b = 0\ and\ a \neq b\}$.

Example:
If $R = Z_5$, then PG(Z_5) is connected graph where V(PG(Z_5) $= \{0,1,2,3,4\}$ and E(PG(Z_5)) $=$ $\{01,02,03,04\}$.

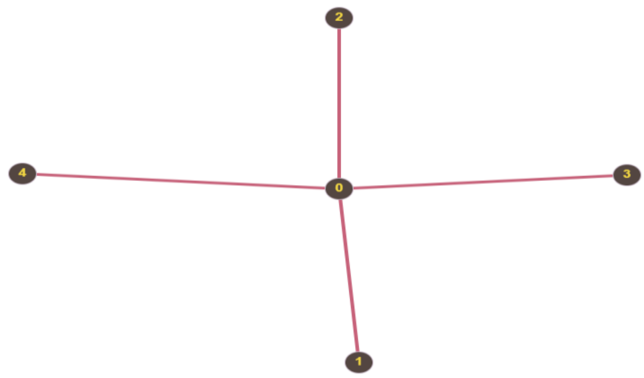

Fig: *2.1 PG*(Z_5)

Remark2.2: Let PG($\boldsymbol{z_p}$) be a graph of R with $R = Z_p$. Then
1. (0) adjacent to all vertices ..
2. A distance between any two distinct vertices a and b is $d(a,b) \leq 2$.

Corollary2.3: If Z_p be a ring , p is a prime number , then PG($\boldsymbol{z_p}$) is a
A star graph.

Corollary2.5: If Z_p be a ring , p is a prime number , then PG($\boldsymbol{z_p}$) is a
1) Connected
2) $\alpha\big(\mathrm{PG}(\boldsymbol{z_p}\big) = 1$

3. Methods and Materials

In this paper, all diagrams are simplfy, bounded, connect, and undirectional. For the graph G =($V(G)$, $E(G)$) let $deg(u)$ be the degree of vertex u in G. If $deg(u)$ = 0, then u is an isolated vertex. Let d (u, v) be the distance between two distinct vertices u and v.

The $ecc(u)$ of the vertex u is $ecc(u)$ = sup $\{d(u, v): v \in V(G)\}$

Next definitions refers to a topological index of a graph G

Eccentricity index of G is [8]

ξ^c ((G))) =$\sum_{u\in V(\mathrm{G})}$ deg(u). e(u)

The Eccentricityty connectivity index of G is [9]

$$X(\mathrm{G}) = \sum_{uv\epsilon E(\mathrm{G})} \frac{1}{\sqrt{\deg(u) \,.\, \deg(v)}}$$

The Sum connectivity index of G is [10]

$$S(\mathrm{G}) = \sum_{uv\epsilon E(\mathrm{G})} \frac{1}{\sqrt{\deg(u) + \deg(v)}}$$

A 1st zagreb index of G is [11]

$$M_1(\mathrm{G}) = \sum_{u\in V(\mathrm{G})} (\deg(u))^2$$

A 2nd zagreb index of G is [11]

$$M_2(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \deg(u) \,.\, \deg(v)$$

The forgotten index of G is [12]

$$F(\mathrm{G}) = \sum_{u\in V(\mathrm{G})} (\deg(u))^3$$

The atomic bond connectivity index of G is [13]

$$Abc(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u) \,.\, \deg(v)}}$$

The Geometric arithmetic index of G is [13]

$$GA(\mathrm{G}) = \sum_{uv\in E(\mathrm{G})} \frac{2\sqrt{\deg(u) \,.\, \deg(v)}}{\deg(u) + \deg(v)}$$

Harmonic index of G is [13]

$$H(\mathrm{G}) = \sum_{uv\in E(\mathrm{G}))} \frac{2}{\deg(u) + \deg(v)}$$

Hosoya polynomial of G is [23]

$$\boldsymbol{H}(G,x) = \sum_{k=0}^{diam(G)} d(G,k)x^k$$

Schults polynomial of G is [24]

$$Sc(G,x) = \sum_{\substack{u,v\epsilon V(G)\\ v\neq u}} (deg(u) + de\,g(v))x^{d(u,v)}$$

3.Resulte and Some Topological indices of PG $(\boldsymbol{z_p})$

NOTE: For every $p \geq 3$, p is prime number then:

$deg(0) = p - 1$, deg$(n) = 1$, $n = 1, \ldots\ldots, p - 1$

$e(0) = 1$, $e(\,n) = 2\,, n = 1, \ldots, p - 1$

Theorem3.1: The eccentrity connectivity index of PG $(\boldsymbol{z_p})$ is

$\xi\left(\text{PG}(\boldsymbol{z_p})\right) = 3p - 3$

Proof:

$$\xi(\text{PG}(\boldsymbol{z_p}) = \sum_{u\in V(\text{PG}(\boldsymbol{z_p}))} deg(u).\,e(u)$$

$$= e(0)\,deg(0) + \underbrace{e(1)\,deg(1) + \ldots\ldots + e(p-1)deg\,(p-1)}_{(p-1)\ times}$$

$$= 3p - 3$$

Theorem3.2: The connectivity index of PG$(\boldsymbol{z_p})$ is $X(\text{PG}(\boldsymbol{z_p}) = \frac{p-1}{\sqrt{\text{p}-1}}$

Proof:

$$X(\text{PG}(\boldsymbol{z_p}) = \sum_{uv\epsilon E(\text{PG}(\boldsymbol{z_p})} \frac{1}{\sqrt{deg(u)\,deg\,(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(0)\,.\,deg\,(1)}} + \cdots + \frac{1}{\sqrt{deg(0)\,.\,deg\,(p-1)}}}_{(p-1)times} +$$

$$= \frac{p-1}{\sqrt{\text{p}-1}}$$

Theorem3.3: : The Sum connectivity index of PG$(\boldsymbol{z_p})$ is

$S(\text{PG}(\boldsymbol{z_p}) = \frac{p-1}{\sqrt{p}}$

Proof:

$$S(\text{PG}(\boldsymbol{z_p}) = \sum_{uv\epsilon E(\text{PG}(\boldsymbol{z_p})} \frac{1}{\sqrt{deg(u) + deg\,(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(0) + deg\,(1)}} + \cdots + \frac{1}{\sqrt{deg(0) + deg\,(p-1)}}}_{(p-1)times}$$

$$= \frac{p-1}{\sqrt{p}}$$

Theorem3.4: The 1st Zagreb index of PG$(\boldsymbol{z_p})$ is

$M_1(\text{PG}(\boldsymbol{z_p}) = (p-1)^2 + (p-1)$

Proof:

$$M_1(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{u\in V(\mathrm{PG}(\boldsymbol{z_p})} (deg(u))^2$$

$$= (deg(0))^2 + \underbrace{(deg(1))^2 + \cdots + (deg(p-1))^2}_{(p-1)\ times}$$

$$= (p-1)^2 + (p-1)$$

Theorem3.5: The second Zagreb index of $\mathrm{PG}(\boldsymbol{z_p})$ is

$M_2(\mathrm{PG}(\boldsymbol{z_p})) = (p-1)^2$

Proof:

$$M_2(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{uv\in E(\mathrm{PG}(\boldsymbol{z_p})} deg(u)\,.\,deg\,(v)$$

$$= \underbrace{deg(0)\,.\,deg(1) + \cdots + deg(0)\,.\,deg\,(p-1)}_{(p-1)times}$$

$$= (p-1)^2$$

Theorem3.6: The forgotten index of $\mathrm{PG}(\boldsymbol{z_p})$ is

$F(\mathrm{PG}(\boldsymbol{z_p})) = (p-1)^3 + (p-1)$

Proof:

$$F(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{u\in V(\mathrm{PG}(\boldsymbol{z_p})} (\deg(\,u))^3$$

$$= (\deg(\,0))^3 + \underbrace{(\deg(\,1))^3 + \cdots + (\deg(\,p-1))^3}_{(p-1)\ times}$$

$$= (p-1)^3 + (p-1)$$

Theorem3.7: The Atom bond connectivity index of $\mathrm{PG}(\boldsymbol{z_p})$ is

$$ABC(\mathrm{PG}(\boldsymbol{z_p})) = (p-1)\sqrt{\frac{p+2}{p-1}}$$

Proof:

$$ABC(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{uv\in E(\mathrm{PG}(\boldsymbol{z_p})} \sqrt{\frac{deg(u) + deg(v) - 2}{deg(u)\,.\,deg\,(v)}}$$

$$= \underbrace{\sqrt{\frac{deg(0) + deg(1) - 2}{deg(0)\,.\,deg\,(1)}} + \cdots + \sqrt{\frac{deg(0) + deg(p-1) - 2}{deg(0)\,.\,deg\,(p-1)}}}_{(p-1)times} += (p-1)\sqrt{\frac{p+2}{p-1}}$$

Theorem3.8: The Geometric-Arithemtic index of $\mathrm{PG}(\boldsymbol{z_p})$ is

$$GA(\mathrm{PG}(\boldsymbol{z_p})) = \frac{2(p-1)\sqrt{\mathrm{p}-1}}{p}$$

Proof:

$$GA(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{uv\in E(\mathrm{PG}(\boldsymbol{z_p}))} \frac{2\sqrt{deg\,(u)\,.\,deg\,(v)}}{deg\,(u) + deg\,(v)}$$

$$= \underbrace{\frac{2\sqrt{deg\,(0)\,.\,deg\,(1)}}{deg\,(0) + deg\,(1)} +..+ \frac{2\sqrt{deg\,(0)\,.\,deg\,(p-1)}}{deg\,(0) + deg\,(p-1)}}_{(p-1)times}$$

$$= \frac{2(p-1)\sqrt{\mathrm{p}-1}}{p}$$

Theorem3.9: The Harmonic index of $\mathrm{PG}(\boldsymbol{z_p})$ is

$$H(\mathrm{PG}(\boldsymbol{z_p})) = \frac{2(p-1)}{p}$$

Proof:

$$H(\mathrm{PG}(\boldsymbol{z_p})) = \sum_{uv \in E(\mathrm{PG}(\boldsymbol{z_p}))} \frac{2}{deg\ (u) + deg\ (v)}$$

$$= \underbrace{\frac{2}{deg\ (0) + deg\ (1)} + \cdots + \frac{2}{deg\ (0) + deg\ (p-1)}}_{(p-1)times}$$

$$= \frac{2(p-1)}{p}$$

Example:

PG (Z_p)	$\xi(PG(Z_p)$	$X(PG(Z_p$	$S(PG(Z$	$M_1(PG($	$M_2(PG($	$F(PG(Z$	$ABC(PG$	$GA(PG(Z_p$	$H(PG(Z_p)$
PG (Z_5)	12	1	$\frac{4}{\sqrt{5}}$	20	16	68	$2\sqrt{7}$	$\frac{16}{5}$	$\frac{8}{5}$
PG (Z_7)	$\frac{7}{18}$	$\frac{6}{\sqrt{6}}$	$\frac{6}{\sqrt{7}}$	55	36	349	3	$\frac{12\sqrt{6}}{7}$	$\frac{12}{7}$

Conclusions:

In this study the definition of Prime graph of a commutative ring (PG(R)) was introduced in chapter two and the number of cycles in (PG (R) was found when. The number of induced complete graph in (PG (R) was computed were. Also, girth and the dominating number of (PG (R) were given.

Discussed and computed the Hosoya polynomial, Schultz polynomial and connectivity index, Zagreb index, forgotten index, sum connectivity index, atom-bond index, geometric index and harmonic index of non-zero graph and the resize graph of twisted group PG(R).

We find some topological indices of prime graph formulas for some degree and eccentricity based topological indices are proposed for Prime graphs of rings Zp, where p is a prime number. To investigate further, examine the prime graph of the ring Zpq, where p, q are prime numbers**.**

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بعض المؤشرات الطوبولوجية للرسم البياني الأولي للحلقة التبديلية Z_p

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معلومات البحث:
تأريخ الاستلام:
تاريخ التعديل:
تأريخ القبــول:
تاريخ الــنشر:
الكلمات المفتاحية:
برايمغراف، مؤشر الاتصال، مجموع مؤشر الاتصال، مؤشر المنسي، مؤشر زغرب الأول.

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الخلاصة:

في هذه الدراسة، تم تقديم تعريف جديد لمفهوم الرسم البياني، وهو الرسم البياني الأولي للحلقة z_p، والذي يُرمز له بـ PG(z_p). تمثل رؤوس الرسم البياني عناصر الحلقة R، بحيث يكون أي رأسين a و b متجاورين بحافة إذا وفقط إذا كان a.b = 0. في هذه الورقة، تم بحث بعض الخصائص الجديدة لـ PG(z_p).