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Some Applications on the Resize Graph for the Twisted Group R (27)

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Abstract

Let G be a group with $2k$ elements, with G-Conjugacy classes set $\{T_1, T_2, \ldots, T_n\}$. The resize graph of G, has a vertex set T, and Ti, Tj are adjacent if there exist involutions $a \in T_i$, $b \in T_j$ such that (ab) again is involution in G. In this pap7er, some application related to topological indices of the resize graph of the Twisted group R (27) group are calculated.

Introduction

Graph theory has become a very well-known and rapidly growing field of mathematics due to its extensive theoretical advances and diverse applications to real-world problems. Although graph theory is still a relatively new field of research, it has yielded many profound and novel discoveries over the past 20 years. In biological, social, physical, and information systems, diagrams can be used to represent a variety of relationships and processes.

A topological index is a real number assigned to the graph whose structure must be a constant. Topological indexes, sometimes is called molecular structure descriptors. More topological indices depend on degrees.
The resize graph and its properties were originally presented by Asaad and Salih [1]. Additional research in this context could be found in [2, 3].

In the a rticles , we study more topological metrics that need degrees of, in (2021) Alaa. J and Akram. S [4] study topological indices and (Schultz and Hosoya) polynomials of the intersection graph of subgroup of the group Zr. The aim of this work is to compute certain topological indices of the resize graph of the Twisted group R (27). Such that the sum connectivity index of $\Gamma^{RS}_{R(27)}$, the first and second zagreb index of $\Gamma^{RS}_{R(27)}$ and other indices.

2. Results

We start by defining certain terms and mentioning a few findings that will be useful in this study. From now on, we may let G the R (27) group. Furthermore, let I(G) be the set of all involution elements in G., and |I(G)|= t. We begin by giving the formula to get the number of edges in the result involution graph

Definition 2.1 [1] Let G be a finite group G, the result involution graph, Γ^{RI}_{G}, is an undirected simple graph that has G, as a vertex set, and two vertices are linked if they are distinct and their product in I(G).

Proposition 2.2[1] Let G be a finite group, and the number of edges in the result involution graph is given by the formula ½ (t |G|-F) such that is F the order of 4 elements of G.

In [5] a resized graph has a vertex set that G-classes, two G-classes X, Y $in\ G$ G, are linked by an edge if X≠Y and their exit x_0 ∈X and y_0∈ Y such that x_0, y_0 are adjacent inΓ^{RI}_{G} . We may denote the resized graph of the group G by Γ^{RS}_{G} . Next the relation betweenΓ^{RI}_{G} and Γ^{RS}_{G} are givev:

Proposition 2.3 [1] In the case of a finite group G. The result involution graph is Γ^{RI}_{G} connected if and only if the resize graph Γ^{RS}_{G} is connected.

Proposition:2.4 [2] For all $v \in$ V(Γ^{RI}_{G}) then:

$$\deg(v) = \begin{cases} t & , \quad otherwise \\ t-1 & , \quad if\ |v| = 4 \end{cases}$$

3.Main Results

In this section, we study some topological indices of $\Gamma^{RS}_{R(27)}$. In case $\Gamma^{RS}_{R(27)}$ there are 35 vertices, where the vertex set of $\boldsymbol{\Gamma^{RS}_{R(27)}}$ are given as:

{ 1A,2A, 3A, 3B,3C,6A, 6B,7A,9A,9B,9C,13A,13B,13C,13D, 13E, 13F, 14A,14B,14C, 19A, 19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E, 37F}. See Figure.1 for deep information on $\Gamma^{RS}_{R(27)}$.

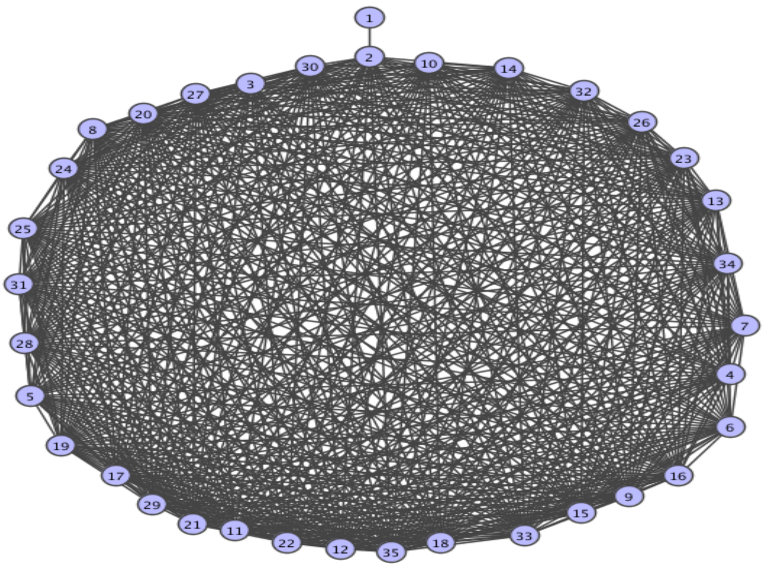

Figure 1. *The resize graph* $\Gamma^{RS}_{R(27)}$ *, G*≅ $R(27)$

Thus, the adjacency of the vertices of $\Gamma^{RS}_{R(27)}$ are presented as follows:

adj(1A)=[2A].
adj(2A)=[1A,3A,7A,9A,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A, 26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F].
adj(3A)=[2A,7A,9A,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F].
adj(3B)=[3C,6B,7A,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B, 26C,26D,26E,26F,37A,37B,37C,37D,37E,37F]. Also, we have **adj**(3C)={**adj**(3B)∪{6A,3B}}\{3C,6B} , **adj**(6A)=**adj**(3B), **adj**(6B)=**adj**(3C).
adj(9B)=[3B,3C,6A,6B,7A,9A,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F]. And, **adj**(9B)=**adj**(9C).
adj(7A)=[2A,3A,3B,3C,6A,6B,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C, 26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F].And, **adj**(X)= {**adj**(7A)∪{7A}}\{X} . for X in {9A,13A,13B,13C,13D, 13E, 13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D, 26E, 26F,37A, 37B,37C,37D,37E, 37F}

By utilizing the above information, we are in good position to give the following theorem:
Theorem 3.1: Certain Topological indices of a graph for $\Gamma = \Gamma^{\boldsymbol{RS}}_{\boldsymbol{R(27)}}$ are presented in the following table:

Table 3.1:Topological indices of a graph G.

Topological Index	**Formula**	**Compute for $\Gamma^{\boldsymbol{RS}}_{\boldsymbol{R(27)}}$**
Eccentricity index of G is [6]	ξ^c (G))) $=\sum_{u\in V(G)} \deg(u).e(u)$	2375
The Eccentricityty connectivity index of G is [7]	$X(G) = \sum_{uv\epsilon E(G)} \frac{1}{\sqrt{\deg(u).\deg(v)}}$	$2.0085055592323 + \frac{\sum_{i=1}^{25}(26-i)}{33}$
The Sum connectivity index of G is [8]	$S(G) = \sum_{uv\epsilon E(G)} \frac{1}{\sqrt{\deg(u)+\deg(v)}}$	$28.7778420224 + \frac{\sum_{i=1}^{25}(26-i)}{\sqrt{66}}$
A 1st zagreb index of G is [9]	$M_1(G) = \sum_{u\in V(G)} (\deg(u))^2$	35550
A 2nd zagreb index of G is [9]	$M_2(G) = \sum_{uv\in E(G)} \deg(u).\deg(v)$	$220091 + \sum_{i=1}^{25}(26-i)(1089)$
The forgotten index of G is [10]	$F(G) = \sum_{u\in V(G)} (\deg(u))^3$	1143580
The atomic bond connectivity index of G is [11]	$Abc(G) = \sum_{uv\in E(G)} \sqrt{\frac{\deg(u)+\deg(v)-2}{\deg(u).\deg(v)}}$	$223.229825927 + \sum_{i=1}^{25}(26-i)\sqrt{\frac{64}{66}}$

TheGeometric arithmetic index of G is [11]	$GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{\deg(u).\deg(v)}}{\deg(u)+\deg(v)}$	$224.58001584+\sum_{i=1}^{25}(26-i)\frac{2\sqrt{1089}}{66}$
Harmonic index of G is [11]	$H(G)=\sum_{uv\in E(G)}\frac{2}{\deg(u)+\deg(v)}$	$7.292163406+\sum_{i=1}^{25}(26-i)\frac{2}{66}$
The reduced second Zagreb index is defined as [12]	$RM_2(G)\sum_{uv\in E(G)}(\deg(u)-1)(\deg(v)-1)$	535254
the Narumi-Katayama index is defined as [13]	$NK(G)=\Pi_{u\in V(G)}\deg(u)$	$2^{6}*3^{33}*5^{4}*7*11^{26}*31^{2}$
the first multiplicative Zagreb index is defined as [12]	$\Pi_1(G)=\pi_{uv\in E(G)}(\deg(u)+\deg(v)$	$(2^{6}*3^{33}*5^{4}*7*11^{26}*31^{2})^{2}$
The second multiplicative Zagreb index is defined as [12]	$\Pi_2(G)=\Pi_{uv\in E(G)}(\deg(u).\deg(v)$	570056
the Randic index [13]	$R(G)=\Pi_{uv\in E(G)}((\deg(u).\deg(v))^{-1}$	0.562764
the Harary index is defined as [14]	$H(G)=\sum_{\{u,v\}\subseteq V(G)}\frac{1}{d(u,v)}$	570.5
The eccentric distance sum index[15]	$\xi^{d}(G)=\sum_{u\in V(G)}e(u)D(u)$	2897

Proof: The calculations of each formula that given in **Table 3.1** mainly depend on the vertex degree and vertex adjacency. Furthermore, this information are already given for $\Gamma_{R(27)}^{RS}$.

Theorem 3.2. The graph $\Gamma_{R(27)}^{RS}$ is connected, such that

TABLE 2. Certain Properties for $\Gamma_{R(27)}^{RS}$

$\lvert E(\Gamma_{R(27)}^{RS})\rvert$	$Diam(\Gamma_{R(27)}^{RS})$	$R(\Gamma_{R(27)}^{RS})$	$\omega(\Gamma_{R(27)}^{RS})$	$\vartheta(\Gamma_{R(27)}^{RS})$	$\chi(\Gamma_{R(27)}^{RS})$
548	3	2	30	3	30

Proof: Since |adj(7A)|=33, and the vertex 1A connected with 7A by 2A. Thus $\Gamma_{R(27)}^{RS}$ is connected. Furthermore. the vertex degrees for $\Gamma_{R(27)}^{RS}$ are respectively equal to:

1, 28, 27, 30, 30, 30, 30, 33, 33, 31, 31, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33.

Therefore, the size of $\Gamma_{R(27)}^{RS}$ is 548. Also, eccentricity of the vertices are given as:

3,2,2,3,3,3,3,2,2,3,3,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2.

As a result, we obtain that $Diam(\Gamma_{R(27)}^{RS})$=3 and $R(\Gamma_{R(27)}^{RS})$=2.

Calculation with in the adjacency of the vertices lead to that $\Gamma_{R(27)}^{RS}$ has six cliques with sizes respectively equal {6,6,6,6,28,2}, we presented as follows:

Cliq1={3C,6A,7A,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F}.

Cliq2={3B,3C,7A,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F}.

Cliq3={6A,6B,7A,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F}.

Cliq4={3B,6B,7A,9A,9B,9C,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F}.

Cliq5={2A,3A,7A,9A,13A,13B,13C,13D,13E,13F,14A,14B,14C,19A,19B,19C,26A,26B,26C,26D,26E,26F,37A,37B,37C,37D,37E,37F}.

Cliq6={1A,2A}

Therefore, $\omega(\Gamma^{RS}_{R(27)})$=30, and evidently that $\vartheta(\Gamma^{RS}_{R(27)})$=3. Now, we aim to prove that $\chi(\Gamma^{RS}_{R(27)})$=30. First we already have $\chi(\Gamma^{RS}_{R(27)})\geq 30$ thus because $\chi(\Gamma^{RS}_{R(27)})\geq \omega(\Gamma^{RS}_{R(27)})$. Thus, we look at the complement of each clique of size 30 individually, and labeled with a color of the same maximum clique:

Cas1: Cliq1^c={1A,2A,3A,3B,6B}, and each vertex of Cliq1^c will labeled as follows:

{1A---37F},{2A---9B},{3A---9C},{3B---6A},{6B---3C}.

Case2: Cliq2^c={1A,2A,3A,6A,6B}, and each vertex of Cliq2^c will labeled as follows:

{1A---37E},{2A---9B},{3A---9C},{6A---3B},{6B---3C}.

Case3: Cliq3^c={1A,2A,3A,3B,3C}, and each vertex of Cliq3^c will labeled as follows:

{1A---37D},{2A---9B},{3A---9C},{3B---6A},{3C---6B}.

Case4:Cliq4^c={1A,2A,3A,3C,6A}, and each vertex of Cliq1^c will labeled as follows:

{1A---37A},{2A---9B},{3A---9C},{3C---6B},{6A---3B}.

Hence, we get $\chi(\Gamma^{RS}_{R(27)})$=30.

Now we can proof the following theorem related to the graph $\Gamma^{RI}_{R(27)}$. By applying the Online Atlas[17] and Gap [16] we can find out the number of the edges between R(27)-classes for the graph $\Gamma^{RI}_{R(27)}$. For example there are 512487 between the classes 1A and 2A. Also, there are 179882937 between the elements of the class 2A. And, there are 373090536 between the class 2A and 3A.

Theorem 3.2. The graph $\Gamma^{RI}_{R(27)}$ is connected 512487-regular graph with girth 3 and size 2581254668560932.

Proof: By using **Theorem 3.1** we have $\Gamma^{RS}_{R(27)}$ is connected thus **Proposition 2.3** implies the connectivity of the graph $\Gamma^{RI}_{R(27)}$. Also, by applying **Proposition 2.4** the graph $\Gamma^{RI}_{R(27)}$ is 512487-regular graph because no elements of order 4 exit in R(27). Moreover, the size of $\Gamma^{RI}_{R(27)}$ is 2581254668560932 by **Proposition 2.2.** Now, since there are 179882937 between the involution elements, thus we can find $\alpha,\beta \in$2A such $\{e,\alpha,\beta\}$ is cycle of length 3, thus the girth is 3.

4.Conclusions

In this study, formulas for some degree and eccentricity based topological indices are proposed for graphs of group R(27), the result involution graphs for the R(27) groups are investigated. In particular, the radius, the diameter, and the girth, as well as full information on resized graphs is provided. This work can be used to study more complicated simple groups, such as monster groups and Pariahs groups.

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بعض التطبيقات لبيان الريسايز لزمرة تويستد R(27)

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الخلاصة:

لتكن G زمرة مع عنصرين 2k مع صفوف تكافؤ $\{T1, T2, \ldots, Tn\}$ فأن بيان الريسايز للزمرة G يملك الرأس v وكل رأسين Tj, Ti, يكونان متجاوران إذا وجد عنصر التفاف a ينتمي للصف Ti وعنصر التفاف b ينتمي للصف Tj بحيث أن ab عنصر التفاف في G. يقدم هذا البحث ايجاد بعض المؤشرات التبولوجية لبيان ناتج الالتفاف لزمرة تويستد R (27) مع متعددات حدود شوالتز وهوسويا.