IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

SJPAS

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

# Reliability Exponential Distribution (1+3) Cascade Model

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https://doi.org/10.54153/sjpas.2026.v8i1.1264

**Article Information**

Received: 20/05/2025
Revised: 27/08/2025
Accepted: 03/09/2025
Published: 10/04/2026

**Keywords:**

*Unit, Monte Carlo, Percentile, Least Squares, and Exponential distribution.*

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**Abstract**

In this paper, a reliability formula was found for a cascade model containing four units (the first unit $\mathcal{D}_1$ is basic and the other three units $\mathcal{D}_2$, $\mathcal{D}_3$, and $\mathcal{D}_4$ are redundant standby). It was assumed that the two variables strength and stress follow the exponential distribution of the variables. The estimation of the exponential distribution parameters was found using three estimation methods (Maximum likelihood, Percentile, and Least Squares), these methods were used to estimate the reliability of the model. MATLAB was used in the Monte Carlo simulation and mean square errs was used to compare the simulation results and find which estimation methods are the best for reliability estimation. The simulation results proved that the estimator ML is the best for estimating the model's reliability.

## 1. Introduction:

The interest in reliability has increased with the increasing dependence on industrial systems, which have become more complex, so it was necessary to pay attention to the life span of these systems and maintain their continued operation for a longer period. Using the mathematical formula $R = (X \geq Y)$ **[1-2]**, reliability can be calculated, where X stands for the strength and Y refers to the for stress, where the unit remains working if X is greater than Y and the unit fails if y becomes greater than X, so many researchers were interested in looking for the possibility of increasing reliability by increasing the strength of in this way, the durability of the new unit is multiplied by the strength attenuation m factor and its interface by Stress attenuation k **[3-5]**.

In this paper, the general formula of reliability will be found for a model containing four units (one main unit and three standby units) where the stress r.v. and strength follow the exponential distribution and reliability estimation is found by methods (ML, Pr, LS), and the results were compared using MSE **[6-8]**.

## 2. Model

In the cascade model (1+3) there are four life periods, and this depends on the sequence of the working unit in the order of the model, as this model consists of four units, namely $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_3$, and $\mathcal{D}_4$ basic units and three standby surplus units, so that when the active unit fails, it is replaced by one of the surplus units, according toorder in the model. The reliability of the model is calculated in four stages, namely $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$ and $\mathcal{S}_4$, and in total, the total reliability of the model R is obtained as follows:

Suppose that random variables for strength and stress follow Exponential distribution where $X \sim \mathrm{Exp}(\beta)$and $Y \sim \mathrm{Exp}(\eta)$ then the CDF of Exp(β) is:

$$F(x) = 1 - e^{-\beta x} \qquad x > 0; \beta > 0 \qquad \text{...(1)}$$

Now, the reliability will be found in each working case of the model and order:

The first stage: the basic unit $\mathcal{D}_1$is active and the rest of the units are surplus, the reliability calculation in this state is as follows:

$$\mathcal{S}_1 = p[X_1 \geq Y_1]$$
$$= \int_0^{\infty} [\overline{F}_{x_1}(y_1)] g(y_1) dy_1$$
$$= \int_0^{\infty} [e^{-\beta_1 y_1}] \eta_1 e^{-\eta_1 y_1} dy_1$$
$$\mathcal{S}_1 = \int_0^{\infty} \eta_1 e^{-(\beta_1+\eta_1) y_1} dy_1$$
$$\mathcal{S}_1 = \left[\frac{\eta_1}{\beta_1+\eta_1}\right] \qquad \text{...(2)}$$

The second stage: in this state, the basic unit $\mathcal{D}_1$ fails and is replaced by unit $\mathcal{D}_{2,}$ and the reliability is calculated as follows:

$$\mathcal{S}_2 = pr[X_1 < Y_1, X_2 \geq Y_2] = pr[X_1 < Y_1, mX_1 \geq kY_1]$$

$$\mathcal{S}_2 = \int_0^{\infty} [F_{x_1}(y_1)] \left[\overline{F}_{x_1}\left(\frac{k}{m} y_1\right)\right] g(y_1) dy_1$$
$$\mathcal{S}_2 = \int_0^{\infty} [1 - e^{-\beta_1 y_1}] \left[e^{-\beta_1 \left(\frac{k}{m}\right) y_1}\right] \eta_1 e^{-\eta_1 y_1} dy_1$$

Let $\frac{k}{m} = \mathcal{Z}$, then

$$\mathcal{S}_2 = \int_0^{\infty} [e^{-\beta_1 \mathcal{Z} y_1} - e^{-\beta_1 (1+\mathcal{Z}) y_1}] \eta_1 e^{-\eta_1 y_1} dy_1$$
$$= \int_0^{\infty} \eta_1 e^{-(\beta_1 \mathcal{Z}+\eta_1) y_1} dy_1 - \int_0^{\infty} \eta_1 e^{-(\beta_1 (1+\mathcal{Z})+\eta_1) y_1} dy_1$$
$$\mathcal{S}_2 = \left(\frac{\eta_1}{\beta_1 \mathcal{Z}+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1 (1+\mathcal{Z})+\eta_1}\right)$$
$$\mathcal{S}_2 = \beta_1 \eta_1 \left[\frac{1}{\beta_1 \mathcal{Z}+\eta_1}\right] \left[\frac{1}{\beta_1 (1+\mathcal{Z})+\eta_1}\right] \qquad \text{...(3)}$$

The third stage: when unit $\mathcal{D}_2$ fails and is replaced by unit $\mathcal{D}_3$, the reliability can be calculated as follows:

$$\mathcal{S}_3 = pr[X_1 < Y_1, X_2 < Y_2, X_3 \geq Y_3]$$
$$\mathcal{S}_3 = pr[X_1 < Y_1, mX_1 < kY_1, mX_2 \geq kY_2]$$
$$\mathcal{S}_3 = pr[X_1 < Y_1, mX_1 < kY_1, m^2 X_1 \geq k^2 Y_1]$$
$$\mathcal{S}_3 = \int_0^{\infty} [F_{x_1}(y_1)] \left[F_{x_1}\left(\frac{k}{m} y_1\right)\right] \left[\overline{F}_{x_1}\left(\frac{k^2}{m^2} y_1\right)\right] g(y_1) dy_1$$
$$\mathcal{S}_3 = \int_0^{\infty} [1 - e^{-\beta_1 y_1}][1 - e^{-\beta_1 \mathcal{Z} y_1}][e^{-\beta_1 \mathcal{Z}^2 y_1}] \eta_1 e^{-\eta_1 y_1} dy_1$$
$$\mathcal{S}_3 = \int_0^{\infty} [1 - e^{-\beta_1 y_1} - e^{-\beta_1 \mathcal{Z} y_1} + e^{-\beta_1 (1+\mathcal{Z}) y_1}][e^{-\beta_1 \mathcal{Z}^2 y_1}] \eta_1 e^{-\eta_1 y_1} dy_1$$
$$\mathcal{S}_3 = \int_0^{\infty} [e^{-\beta_1 \mathcal{Z}^2 y_1^{\sigma}} - e^{-\beta_1 (1+\mathcal{Z}^2) y_1^{\sigma}} - e^{-\beta_1 (\mathcal{Z}+\mathcal{Z}^2) y_1^{\sigma}} + e^{-\beta_1 (1+\mathcal{Z}+\mathcal{Z}^2) y_1^{\sigma}}]$$
$$. \eta_1 e^{-\eta_1 y_1} dy_1$$
$$\mathcal{S}_3 = \int_0^{\infty} \eta_1 e^{-(\beta_1 \mathcal{Z}^2+\eta_1) y_1} dy_1 - \int_0^{\infty} \eta_1 e^{-(\beta_1 (1+\mathcal{Z}^2)+\eta_1) y_1} dy_1$$

$$-\int_0^\infty \eta_1 e^{-(\beta_1(\mathcal{Z}+\mathcal{Z}^2)+\eta_1)y_1}dy_1 + \int_0^\infty \eta_1 e^{-(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2)+\eta_1)y_1}dy_1$$

$$\mathcal{S}_3 = \left(\frac{\eta_1}{\beta_1\mathcal{Z}^2+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(1+\mathcal{Z}^2)+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(\mathcal{Z}+\mathcal{Z}^2)+\eta_1}\right) + \left(\frac{\eta_1}{\beta_1(1+\mathcal{Z}+\mathcal{Z}^2)+\eta_1}\right)$$

$$\mathcal{S}_3 = 2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1\mathcal{Z}^2+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}^2)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^2)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2)+\eta_1)}\right] \quad ...(4)$$

The fourth stage: when unit $\mathcal{D}_3$ fails and is replaced by unit $\mathcal{D}_4$, reliability is calculated as follows:

$$\mathcal{S}_4 = \mathrm{pr}[X_1 < Y_1, X_2 < Y_2, X_3 < Y_3, X_4 \geq Y_4]$$

$$\mathcal{S}_4 = \mathrm{pr}[X_1 < Y_1, m X_1 < k Y_1, m^2 X_1 < k^2 Y_1, m^3 X_1 \geq k^3 Y_1]$$

$$\mathcal{S}_4 = \int_0^\infty [F_{x_1}(y_1)]\left[F_{x_1}\left(\frac{k}{m}y_1\right)\right]\left[F_{x_1}\left(\frac{k^2}{m^2}y_1\right)\right]\left[\overline{F}_{x_1}\left(\frac{k^3}{m^3}y_1\right)\right]g(y_1)dy_1$$

$$\mathcal{S}_4 = \int_0^\infty [1-e^{-\beta_1 y_1}][1-e^{-\beta_1\mathcal{Z}y_1}][1-e^{-\beta_1\mathcal{Z}^2y_1}][e^{-\beta_1\mathcal{Z}^3y_1}]\,\eta_1 e^{-\eta_1 y_1}dy_1$$

$$\mathcal{S}_4 = \int_0^\infty [1-e^{-\beta_1 y_1}-e^{-\beta_1\mathcal{Z}y_1}+e^{-\beta_1(1+\mathcal{Z})y_1}][1-e^{-\beta_1\mathcal{Z}^2y_1}][e^{-\beta_1\mathcal{Z}^3y_1}]\eta_1 e^{-\eta_1 y_1}dy_1$$

$$\mathcal{S}_4 = \int_0^\infty [1-e^{-\beta_1 y_1}-e^{-\beta_1\mathcal{Z}y_1}+e^{-\beta_1(1+\mathcal{Z})y_1}-e^{-\beta_1\mathcal{Z}^2y_1}+e^{-\beta_1(1+\mathcal{Z}^2)y_1}+e^{-\beta_1(\mathcal{Z}+\mathcal{Z}^2)y_1}$$
$$-e^{-\beta_1(1+\mathcal{Z}+\mathcal{Z}^2)y_1}][e^{-\beta_1\mathcal{Z}^3y_1}]\;\eta_1 e^{-\eta_1 y_1}dy_1$$

$$= \int_0^\infty [e^{-\beta_1\mathcal{Z}^3y_1}-e^{-\beta_1(1+\mathcal{Z}^3)y_1}-e^{-\beta_1(\mathcal{Z}+\mathcal{Z}^3)y_1}+e^{-\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)y_1}-e^{-\beta_1(\mathcal{Z}^2+\mathcal{Z}^3)y_1}$$
$$+e^{-\beta_1(1+\mathcal{Z}^2+\mathcal{Z}^3)y_1}+e^{-\beta_1(\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)y_1}-e^{-\beta_1(1+\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)y_1}]\eta_1 e^{-\eta_1 y_1}dy_1$$

$$\mathcal{S}_4 = \int_0^\infty \eta_1 e^{-(\beta_1\mathcal{Z}^3+\eta_1\rho_1)y_1}dy_1 - \int_0^\infty \eta_1 e^{-(\beta_1(1+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1$$
$$-\int_0^\infty \eta_1 e^{-(\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1 + \int_0^\infty \eta_1 e^{-(\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1$$
$$-\int_0^\infty \eta_1 e^{-(\beta_1(\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1 + \int_0^\infty \eta_1 e^{-(\beta_1(1+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1$$
$$+\int_0^\infty \eta_1 e^{-(\beta_1(\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1 - \int_0^\infty \eta_1 e^{-(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1\rho_1)y_1}dy_1$$

$$\mathcal{S}_4 = \left(\frac{\eta_1}{\beta_1\mathcal{Z}^3+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(1+\mathcal{Z}^3)+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1}\right) + \left(\frac{\eta_1}{\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1}\right)$$
$$+\left(\frac{\eta_1}{(\beta_1(1+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1)}\right) + \left(\frac{\eta_1}{\beta_1(\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1}\right) - \left(\frac{\eta_1}{\beta_1(1+\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1}\right)$$

$$\mathcal{S}_4 = 2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1\mathcal{Z}^3+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]$$
$$-2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1)}\right] \quad ...(5)$$

Finally, by combining the above four states, the general formula of reliability of the (3+1) cascade model is obtained:

$$R = \left[\frac{\eta_1}{\beta_1+\eta_1}\right] + \beta_1\eta_1\left[\frac{1}{\beta_1\mathcal{Z}+\eta_1}\right]\left[\frac{1}{\beta_1(1+\mathcal{Z})+\eta_1}\right] + 2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1\mathcal{Z}^2+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}^2)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^2)+\eta_1)}\right]$$
$$\cdot\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2)+\eta_1)}\right] + 2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1\mathcal{Z}^3+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]$$
$$-2\eta_1\beta_1^2\mathcal{Z}\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1)}\right]\left[\frac{1}{(\beta_1(1+\mathcal{Z}+\mathcal{Z}^2+\mathcal{Z}^3)+\eta_1)}\right] \quad ...(6)$$

## 3. Estimation:

The exponential distribution parameters are estimated using three different estimation methods as follows:

### 3-1Maximum likelihood

Let the random sample $x_1, x_2, \dots, x_n$ from Exp(β), the likelihood function "L", is **[9-11]**:

$$L(x_1, x_2, \dots, x_n, \beta) = f(x_1;\beta)f(x_2;\beta)\dots f(x_n;\beta)$$
$$= \prod_{i=1}^{n} f(x_n;\beta) \quad ...(7)$$

So the equation 7 becomes:

$$L(x_1, x_2, \dots, x_n; \beta) = \beta^n \prod_{i=1}^{n} e^{-\sum_{i=1}^{n} \beta x} \quad \text{...(8)}$$

Equation 8 becomes after taking its logarithm and deriving it as written below

$$\ln L = nL n\beta - \beta \sum_{i=1}^{n} x$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x$$

$$\frac{n}{\beta} - \sum_{i=1}^{n} x = 0$$

So the maximum likelihood estimator of $\beta$:

$$\hat{\beta}_{ML} = \frac{n}{\sum_{i=1}^{n} x} \quad \text{...(9)}$$

And

$$\hat{\eta}_{ML} = \frac{m}{\sum_{j=1}^{m} y} \quad \text{...(10)}$$

**3-2 Percentile Method**

Let $x_i$ ; $i = 1,2, \dots, n$ of $\text{Exp}(\beta)$, by using equation 1, **[12]**:

$$\ln\left(1 - F\left(x_{(i)}\right)\right) = -\beta x_{(i)}$$

$$x_{(i)} = \left(\frac{-\ln\left(1 - F\left(x_{(i)}\right)\right)}{\beta}\right) \quad \text{...(11)}$$

$$Ln[(1 - P_i)^{-1}] = \beta x_{(i)} \quad \text{...(12)}$$

Where $P_i$ is plotting position and $P_i = \frac{i}{n+1}$ ; $i = 1,2, \dots, n$

$$x_{(i)} = \left(\frac{-\ln(1 - P_i)}{\beta}\right) \quad \text{...(13)}$$

The minimizing equation is:

$$\sum_{i=1}^{n}\left[x_{(i)} - F\left(x_{(i)}\right)\right]^2 \quad \text{...(14)}$$

Substitution 13 in 14, then:

$$\sum_{i=1}^{n}\left[x_{(i)} - \left(\frac{-\ln(1 - P_i)}{\beta}\right)\right]^2 \quad \text{...(15)}$$

By deriving the equation 15:

$$\sum_{i=1}^{n} 2\left[\left(x_{(i)}\right) - \beta^{-1}(-\ln(1 - P_i))\right](\beta^{-2})(-\ln(1 - P_i)) = 0$$

So that:

$$\hat{\beta}_{Pr} = \left[\frac{\sum_{i=1}^{n}(-\ln(1 - P_i))^2}{\sum_{i=1}^{n}\left(x_{(i)}\right)(-\ln(1 - P_i))}\right] \quad \text{...(16)}$$

and

$$\hat{\eta}_{Pr} = \left[\frac{\sum_{j=1}^{m}(-\ln(1 - P_j))^2}{\sum_{j=1}^{m}\left(y_{(j)}\right)(-\ln(1 - P_j))}\right] \quad \text{...(17)}$$

**3-1 Least Squares Method**

Let $x_1, x_2, \dots, x_n$ with $\text{Exp}(\beta)$, the equation is used as follow, **[13-16]**:

$$Q = \sum_{i=1}^{r}\left(\hat{F}\left(x_{(i)}\right) - F\left(x_{(i)}\right)\right)^2$$

$$= \sum_{i=1}^{n}\left(\hat{F}\left(x_{(i)}\right) - \left(1 - e^{-\beta x}\right)\right)^2 \quad \text{...(18)}$$

So that:

$$-\ln\left(1 - F\left(x_{(i)}\right)\right) = \theta x_{(i)}^{\sigma} \quad \text{...(19)}$$

Let $q_{(i)} = -\ln\left(1 - \hat{F}\left(x_{(i)}\right)\right) = -\ln(1 - P_i)$

Then Equation 19 becomes:

$S(2,\lambda) = \sum_{i=1}^{n}\left(q_{(i)} - \beta x_{(i)}\right)^2$ ...(20)

So that:

$\frac{\partial S(\beta)}{\beta} = \sum_{i=1}^{n} 2\left(q_{(i)} - \beta x_{(i)}\right)\left(-x_{(i)}\right) = 0$

$-\sum_{i=1}^{n} q_{(i)}\, x_{(i)} + \beta \sum_{i=1}^{n} x_{(i)}^2 = 0$

Then $\hat{\beta}_{LS}$ :

$\hat{\beta}_{LS} = \frac{\sum_{i=1}^{n} q_{(i)}\, x_{(i)}}{\sum_{i=1}^{n} x_{(i)}^2}$ ...(21)

and

$\hat{\eta}_{LS} = \frac{\sum_{j=1}^{m} q_{(j)}\, y_{(j)}}{\sum_{j=1}^{m} y_{(j)}^2}$ ...(22)

**4. The simulation**

Using the estimates obtained, a simulation is made to compare results of these estimates using MSE. Different small, medium, and large sample sizes were used, and different parameter values. Six experiments were conducted for various parameter values to compare the estimation results. Experiments made were founded on run size K=5000. Sample sizes are $(n, m) = (20, 20), (40, 40), and\ (80, 80)$, where $\mathcal{N}_1 = (20, 20)$, $\mathcal{N}_2 = (40, 40), and\ \mathcal{N}_3 = (80, 80)$. The values of sex experiments are included in Table 1:

**Table 1:** The experiment values.

| experiment | $k$ | $m$ | $\beta_1$ | $\eta_1$ | $R$ |
|---|---|---|---|---|---|
| 1 | 1.80 | 0.30 | 0.7 | 0.7 | 0.5179 |
| 2 | 1.20 | 0.80 | 0.7 | 0.7 | 0.6287 |
| 3 | 1.80 | 0.30 | 0.9 | 0.7 | 0.4523 |
| 4 | 1.20 | 0.80 | 0.9 | 0.7 | 0.5524 |
| 5 | 1.80 | 0.30 | 0.7 | 1.5 | 0.7106 |
| 6 | 1.20 | 0.80 | 0.7 | 1.5 | 0.8209 |

The results of the simulation for experiments:

**Table 2**: Simulation of experiment (1)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.5192 | 0.5194 | 0.5195 | |
| | MSE | 0.0068 | 0.0076 | 0.0082 | ML |
| $\mathcal{N}_2$ | Mean | 0.5183 | 0.5179 | 0.5177 | |
| | MSE | 0.0034 | 0.0040 | 0.0043 | ML |
| $\mathcal{N}_3$ | Mean | 0.5187 | 0.5190 | 0.5191 | |
| | MSE | 0.0017 | 0.0020 | 0.0022 | ML |

**Table 3:** Simulation of experiment (2)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.6248 | 0.6262 | 0.5950 | |
| | MSE | 0.0083 | 0.0092 | 0.0099 | ML |
| $\mathcal{N}_2$ | Mean | 0.6282 | 0.6294 | 0.5987 | ML |

| | | | | | |
|---|---|---|---|---|---|
| | MSE | 0.0041 | 0.0048 | 0.0055 | |
| $\mathcal{N}_3$ | Mean | 0.6290 | 0.6291 | 0.5988 | |
| | MSE | 0.0022 | 0.0025 | 0.0033 | ML |

**Table 4:** Simulation of experiment (3)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.5190 | 0.5193 | 0.5193 | |
| | MSE | 0.0111 | 0.0119 | 0.0125 | ML |
| $\mathcal{N}_2$ | Mean | 0.5172 | 0.5172 | 0.5175 | |
| | MSE | 0.0077 | 0.0082 | 0.0086 | ML |
| $\mathcal{N}_3$ | Mean | 0.5183 | 0.5186 | 0.5188 | |
| | MSE | 0.0062 | 0.0065 | 0.0066 | ML |

**Table 5:** Simulation of experiment (4)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.6222 | 0.6235 | 0.5991 | |
| | MSE | 0.0110 | 0.0142 | 0.0121 | ML |
| $\mathcal{N}_2$ | Mean | 0.6250 | 0.6254 | 0.6014 | |
| | MSE | 0.0095 | 0.0101 | 0.0171 | ML |
| $\mathcal{N}_3$ | Mean | 0.6247 | 0.6250 | 0.6016 | |
| | MSE | 0.0074 | 0.0078 | 0.0089 | ML |

**Table 6**: Simulation of experiment (5)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.5174 | 0.5172 | 0.5171 | |
| | MSE | 0.0443 | 0.0452 | 0.0457 | ML |
| $\mathcal{N}_2$ | Mean | 0.5189 | 0.5191 | 0.5192 | |
| | MSE | 0.0402 | 0.0406 | 0.0409 | ML |
| | Mean | 0.5185 | 0.5182 | 0.5182 | |
| $\mathcal{N}_3$ | MSE | 0.0386 | 0.0390 | | |
| | | | | 0.0392 | ML |

**Table 7:** Simulation of experiment (6)

| S.S. | Create | ML | Pr | LS | Favorite |
|---|---|---|---|---|---|
| $\mathcal{N}_1$ | Mean | 0.8209 | 0.6267 | 0.5953 | |
| | MSE | 0.0466 | 0.0470 | 0.0597 | ML |
| $\mathcal{N}_2$ | Mean | 0.6274 | 0.6279 | 0.5969 | |
| | MSE | 0.0417 | 0.0422 | 0.0549 | ML |
| $\mathcal{N}_3$ | Mean | 0.6293 | 0.6298 | 0.5994 | |
| | MSE | 0.0389 | 0.0390 | 0.0514 | ML |

## 5. Conclusions

After conducting the six experiments, it was concluded that the parameter values affect the reliability values of the model. When the value of the parameter β increases, the reliability of the model decreases, while the value of the parameter increases with the value of the parameter η, and the reliability value decreases with an increase of $\frac{k}{m}$. This is evident when comparing the reliability values in Table 1.

After conducting simulations for the six experiments and for different sample sizes, it was concluded that estimator ML is the best for estimating the model's reliability and for all the results, as is clear in Tables 2 to 7.

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www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

# معولية التوزيع الاسي لنموذج كاسكاد (3+1)

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**معلومات البحث:**

تأريخ الاستلام:20/05/2025
تاريخ التعديل: 27/08/2025
تأريخ القبـــول: 03/09/2025
تاريخ الـنشر: 10/04/2026

**الكلمات المفتاحية:**

*الكلمة المفتاحية: الوحدة، مونت كارل، النسبة المؤوية ،المربعات الصغرى ، التوزيع الاسي*

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**الخلاصة:**

في هذه البحث، تم إيجاد صـــيغة المعولية لنموذج متتالي يحتوي على أربع وحدات (الوحدة الأولى الاساسية $\mathcal{D}_1$ والوحدات الثلاث الأخرى $\mathcal{D}_2$ و $\mathcal{D}_3$ و $\mathcal{D}_4$ احتياطية زائدة). تم افتراض أن متغيري المتانة والإجهاد يتبعان التوزيع الأسي. تم إيجاد تقدير معلمات التوزيع الأسي باستخدام ثلاث طرق تقدير هي (الإمكان الاعظم، النسبة المئوية، والمربعات الصغرى) ومن ثم تم تقدير موثوقية النموذج بهذه الطرق. تم إجراء محاكاة مونت كارلو باستخدام برنامج MATLAB لمقارنة نتائج طرق التقدير ومعرفة أيهما أفضل لتقدير موثوقية النموذج باستخدام متوسط مربع الخطأ وقد وجد أن مقدر الإمكان الأعظم هو الأفضل لتقدير معولية النموذج.