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Monte Carlo Investigation of Phase Transitions in the Ising Model under Binary Quenched Bond Disorder

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Abstract

The effect of quenched disorder on the critical behavior of the two-dimensional Ising model has been systematically investigated using extensive Monte Carlo simulations and finite-size scaling analysis. To verify the distinctive critical exponents of the pure universality class, the clean 2D Ising system was initially examined. A binary distribution of exchange couplings with different disorder concentrations was then used to introduce disorder, enabling a thorough analysis of its effects on the scaling characteristics close to the critical temperature. Thermodynamic parameters such as correlation length, magnetization, and susceptibility are analyzed, and the results show that disorder contributes significant logarithmic corrections to the scaling laws while the leading critical exponents stay close to those of the clean Ising model. These findings verify that, despite disorder-dependent logarithmic changes to its scaling behavior, the quenched binary Ising model maintains the Ising universality class. Further evidence that weak disorder in two dimensions causes marginally irrelevant perturbations that provide logarithmic corrections to critical scaling is provided by the current results, which are in agreement with theoretical expectations from renormalization-group analysis and earlier numerical investigations.

Introduction

In modern statistical mechanics and condensed matter physics, the study of phase transitions and critical phenomena remains a fundamental area of research. Understanding how macroscopic order arises from microscopic interactions is crucial for grasping cooperative behavior across various systems, including brain networks, social models, and magnetic materials [1, 2]. Due to its simplicity, mathematical elegance, and universality, the Ising model occupies a prominent position among the theoretical frameworks developed to explain such collective phenomena. The Ising model, which has become a benchmark for testing theories of criticality and scaling, effectively represents essential features of ferromagnetic ordering and continuous phase transitions, despite its straightforward formulation [3].

The square-lattice Ising model holds great importance in two dimensions due to Onsager's precise solution, which confirmed the existence of a finite-temperature phase transition and provided accurate critical exponents for a system characterized by short-range interactions [4]. Since then, this model has served as a benchmark for both analytical and computational techniques. Nevertheless, real materials are seldom perfect; various forms of disorder arise from impurities, defects in the lattice, and compositional irregularities, which can greatly influence their thermodynamic and critical characteristics [5]. Therefore, a key focus within statistical physics and materials science is to explore how disorder affects the nature and universality of phase transitions.

The Harris criterion [6], which assesses the relevance of randomness in a system's essential behavior based on the sign of the specific heat exponent α in the pure model, serves as a key theoretical framework for understanding the effects of disorder. In the case of the two-dimensional Ising model, $\alpha = 0$ suggests that disorder is only minimally relevant, which could lead to logarithmic scaling corrections without altering the universality class [7, 8]. This has led to a significant amount of theoretical and numerical research on disordered Ising systems, aimed at confirming the accuracy of these predictions and identifying the type of corrections caused by disorder.

Quenched disorder is one of the numerous types of disorders [9]. Local couplings or site occupancies are examples of random variables that remain constant during thermal equilibration in quenched systems and represent impurities or compositional changes that are static on the timescale of spin fluctuations [10]. One well-researched example of such a disorder is the random-bond Ising model (RBIM), in which the degree of interaction between nearby spins is randomly distributed according to a certain probability law. "Strong" and "weak" bonds are the two potential coupling values that each bond in the binary quenched bond disorder scenario has a fixed probability of adopting. Because it encapsulates the basic physics of disordered ferromagnets, amorphous alloys, and composite magnetic materials and is computationally tractable, this model is especially appealing [11].

Over the past 20 years, significant advancements in simulation techniques and computer power have enabled in-depth numerical analysis of the RBIM. Monte Carlo (MC) simulations using efficient methods like as Metropolis, Swendsen–Wang, and Wolff cluster updates have been widely used to study critical features, finite-size scaling, and disorder averaging in the two-dimensional [12, 13, 14]. According to recent research, bond randomness provides observable finite-size effects and logarithmic corrections, but the critical exponents are still consistent with the pure Ising universality class [15, 16]. These small differences have been verified using conformal field theory methods, large-scale simulations, and renormalization-group analysis.

The impact of quenched disorder on the universality class of the Ising system has been the subject of numerous analytical investigations using renormalization group (RG) and computational techniques. Weak random-bond disorder in 2D causes logarithmic divergences in the specific heat and susceptibility, as first shown by Dotsenko and Dotsenko [17]. Ludwig [18] improved this finding by obtaining explicit expressions for the logarithmic terms. In agreement with RG predictions, subsequent numerical studies by Lessa et al. [19] and Gordillo-Guerrero et al. [20] offered compelling evidence that the disordered 2D Ising model exhibits logarithmic corrections in thermodynamic quantities while retaining the critical exponents of the pure system. The universality of these logarithmic factors has been proven by more recent high-precision Monte Carlo simulations (Gluth et al. [21]; Letouzé et al. [22]), which have also shown that the random-bond and random-site versions exhibit similar scaling behavior within numerical accuracy.

The majority of earlier research has either concentrated on weak disorder regimes or taken into account a small range of disorder amplitudes, even though the general image of marginal irrelevance is strongly supported. As a result, the precise relationship between logarithmic corrections and the strength of quenched binary disorder remains unclear. Specifically, it has not been well examined how growing disorder affects the behavior of thermodynamic parameters such as magnetization, susceptibility, and specific heat. In this study, we employ extensive Monte Carlo simulations and finite-size scaling analysis to fill this gap by methodically examining how the thermodynamic behavior of the clean two-dimensional Ising model is affected by the strength of quenched binary disorder. We investigate the evolution of the critical scaling of these values and whether the logarithmic corrections suggested by RG theory get strengthened or modified with increased randomness by altering the disorder amplitude. This approach advances our knowledge of disorder-induced phenomena in low-dimensional systems by shedding some light on the stability of the marginal irrelevance scenario and elucidating the crossover between weak- and strong-disorder regimes.

Model and theory

As a clean reference system, the two-dimensional Ising model with nearest-neighbor ferromagnetic contacts is used to investigate the effect of quenched disorder strength on the universality class of the phase transition. This model consists of a square lattice with dimensions $N = L \times L$, where a classical spin variable, $S_i = \pm 1$, occupies each lattice site i. There is no external magnetic field present. The following represents the clean (pure) ferromagnetic Ising model's Hamiltonian[4]:

$$H_{\text{clean}} = -J \sum_{\langle i,j \rangle} S_i S_j, \qquad (1)$$

where $J > 0$ is the uniform exchange interaction favoring parallel spin alignment, and the summing $\langle i, j \rangle$ runs over all nearest-neighbor pairings. The Boltzmann distribution controls the likelihood of a spin configuration $\{S_i\}$ at thermal equilibrium:

$$P(\{S_i\}) = \frac{1}{Z} e^{-\beta H}, \qquad (2)$$

where $\beta = 1/(k_B T)$, k_B is the Boltzmann constant, and Z is the partition function:

$$Z = \sum_{\{S_i\}} e^{-\beta H}. \qquad (3)$$

At low temperatures, the clean system experiences a continuous (second-order) phase transition from a magnetically ordered (ferromagnetic) phase to a disordered (paramagnetic) phase at high temperatures. According to Onsager [4], the precise critical temperature for the two-dimensional Ising model is:

$$k_B T_c = \frac{2J}{\ln(1+\sqrt{2})} \approx 2.269J. \qquad (4)$$

Below T_c, the system develops a spontaneous magnetization $M(T)$, given by the Onsager–Yang relation [23]:

$$M(T) = \begin{cases} [1 - \sinh^{-4}(2J/k_B T)]^{1/8}, & T < T_c \\ 0 & , \quad T \geq T_c. \end{cases} \qquad (5)$$

Onsager's exact solution predicts the following exponent values for the pure 2D Ising universality class: specific heat exponent $\alpha = 0$, correlation length exponent $\nu = 1$, susceptibility exponent $\gamma = 7/4$, and magnetization exponent $\beta = 1/8$ [2]. These precise findings offer a useful starting point for examining the impact of disorder. We incorporate binary quenched bond disorder in the nearest-neighbor coupling constants to study the impact of quenched impurities on the critical behavior. The exchange coupling J_{ij} between spins S_i and S_j is no longer uniform in this disordered version, but rather is given at random using a binary probability distribution [6, 24]:

$$P(J_{ij}) = p\,\delta(J_{ij} - J_1) + (1-p)\,\delta(J_{ij} - J_2), \qquad (6)$$

where $J_1 > J_2 > 0$ denote the strong and weak coupling constants, respectively, p is the concentration of strong bonds. The Hamiltonian of the disordered system becomes:

$$H_{\text{dis}} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j. \qquad (7)$$

The set of random couplings $\{J_{ij}\}$ stays constant during the thermal averaging of the spin degrees of freedom because the disorder is quenched. The dimensionless ratio can be used to measure the strength of disorder [25]:

$$r = \frac{J_2}{J_1}, \quad 0 < r \leq 1, \qquad (8)$$

where $r = 1$ recovers the clean system, while smaller values of r correspond to increasing disorder. The Harris criterion [3] states that the specific-heat exponent α of the clean system determines how quenched chaos affects crucial behavior. If $\alpha > 0$, the disorder is relevant; if $\alpha < 0$, it is irrelevant. $\alpha = 0$ for the 2D Ising model, this places it in a marginal situation, where disorder might cause scaling corrections that are logarithmic rather than completely changing the universality class. The Ising critical exponents for weak bond randomness are generally constant, although observables like the magnetization and susceptibility display logarithmic singularities rather than pure power laws, according to numerical studies and renormalization group (RG) investigations [26, 27]. The critical temperature may also be somewhat shifted by the disorder, resulting in an effective transition temperature $T_c(p, J_1, J_2)$.

Computational method

When exact analytical solutions are either unavailable or limited to specific situations, Monte Carlo simulations provide a reliable computational approach for studying statistical mechanical systems [28–30]. Onsager's exact solution for the two-dimensional Ising model can be found at $J_1 = J_2 = 1$. For investigating finite-size effects, confirming universality, and extending the analysis to variations in disorder strength, numerical simulations remain essential. Algorithms that flip single spins, such as the Metropolis technique [31, 32], suffer from critical slowing down at temperatures around the critical point, which causes diverging correlation times and traps the system in local spin states. To solve this problem, we employ the Wolff cluster method [13, 14], which significantly reduces autocorrelation times by jointly updating groups of correlated spins. This study investigates the critical behavior of the clean 2D square-lattice Ising model for nearest-neighbor interactions using large-scale Monte Carlo simulations in conjunction with the Wolff cluster algorithm. We model systems with periodic boundary conditions that range in size from 20^2 to 500^2 sites, and we modify the binary quenched bond disorder strength between $r = 0.25, 0.50, 0.75, \text{and } 1.00$ with impurity concentration $p = 0.5$. Several simulations are carried out at different temperatures close to the expected critical point for each system size L and r. Every result is averaged over a wide range of disorder configurations. Each sample is equilibrated using 400 full Monte Carlo sweeps, and observables are measured using 2000 sweeps (one measurement per sweep). Thermodynamic properties like magnetization M, energy E, magnetic susceptibility χ, Specific heat $C_{v,}$ and Binder cumulant U have been computed from the following quantities [30, 33]:

$$\langle M\rangle = \frac{1}{N}\sum_{i}^{N} S_i, \tag{9}$$

$$\langle E\rangle = -J\frac{1}{N}\sum_{<ij>} S_i S_j\,, \tag{10}$$

$$\chi = \frac{1}{NK_B T}\left(\langle M^2\rangle - \langle M\rangle^2\right)\ , \tag{11}$$

$$C_v = \frac{1}{NK_B T^2}\left(\langle E^2\rangle - \langle E\rangle^2\right)\ , \tag{12}$$

$$U = 1 - \frac{\langle M^4\rangle}{3\langle M^2\rangle^2}\ . \tag{13}$$

We use the finite-size scaling theory to obtain crucial exponents. In the vicinity of criticality, the correlation length exhibits $\xi\sim\mid T-Tc\mid^{-\nu}$. The maximum correlation length that can exist for a finite system of size L is constrained by L. We can match them $L\sim\xi\ \sim\mid T-Tc\mid^{-\nu}$ at pseudo-criticality, and rearranging results in:

$$\frac{1}{L}\sim|T-T_C|^{\nu}. \tag{14}$$

Additionally, around the critical temperature T_C, thermodynamic parameters as the magnetization M, susceptibility χ, and specific heat C_v exhibit single behavior [34]:

$$M(t,L)\sim L^{-\beta/\nu}, \tag{15}$$

$$\chi(t,L)\sim L^{\gamma/\nu}\,, \tag{16}$$

$$C_v(t,L)\sim lnL\,, \tag{17}$$

where $t = (T - T_C)\ /\ T_C$ is the reduced temperature.

However, multiplicative logarithmic factors that change these forms are introduced in the disordered case due to marginally meaningful randomness [7, 35]:

$$\frac{1}{L}\sim|T-T_C|^{\nu}|\ln|T-T_C||^{\tilde{\nu}}, \tag{18}$$

$$M(t,L)\sim L^{-\beta/\nu}\,(\ln L)^{-\tilde{\beta}}, \tag{19}$$

$$\chi(t,L)\sim L^{\gamma/\nu}(\ln L)^{\tilde{\gamma}}. \tag{20}$$

Both the leading critical exponents (β, γ, ν) and the logarithmic correction exponents ($\tilde{\beta}, \tilde{\gamma}, \tilde{\nu}$) (the strength of the logarithmic corrections) can be extracted by fitting the finite-size dependency of these equations. This method allows a quantitative assessment of whether the critical behavior shows minor disorder-induced corrections or stays in the pure Ising universality class.

Results and discussion

Clean Ising model

The clean two-dimensional square-lattice Ising model for nearest-neighbor spin interactions is first examined, with the spin–spin interaction strength set to $J_1 = J_2 = 1$ ($r = 1.0$). This is accomplished by using periodic boundary conditions and the extended Monte Carlo approach on systems ranging in size from 20 to 500. The behavior of the magnetization M, susceptibility χ, and specific heat C_v are used to estimate the critical temperature T_C. The impact of finite-size effects is clear in Figure 1.

As shown in Figure 1a—c, a steady decrease in magnetization, accompanied by the existence of finite, widened peaks in susceptibility and specific heat, is an indicator of the phase transition. The observed transition sites (the twists in the magnetization curve or the locations of the peaks in χ and C_v) depend on L and do not precisely correspond to the critical temperature because these values are computed on finite lattices of size L. The peaks progressively approach the precise Onsager value $T_C = 2.269J/kB$ [4] as the lattice size grows, but full convergence is only attained in the thermodynamic limit ($L \rightarrow \infty$). As a result, although the graphs confirm that the phase transition occurred, they are not enough to determine the precise value of T_C; instead, techniques of finite-size scaling and extrapolation of the pseudocritical temperatures $T_C(L)$ are required to determine the precise critical temperature [16].

We employ the Binder cumulant technique to deal with the finite-size inconsistencies [8]. The Binder cumulant is calculated for different system dimensions L and is explained in Eq. 7. The critical temperature ($T_C = 2.269J/kB$, which fully agrees with Ref. [4]) can be precisely and size-independently estimated from the point where the $U(T)$ curves intersect for increasing values of L (see Fig. 2). The uncertainty associated with determining Tc based only on the peaks in susceptibility and heat capacity is significantly reduced by this method. Once we have approximate crossing spots, we can also use finite-size extrapolation to improve the estimation of T_C. Thus, the incorporation of Binder cumulant crossings circumvents the limitations imposed by finite lattice sizes and enables a reliable evaluation of the critical temperature. After obtaining a reliable estimate of the critical temperature, we proceeded to determine the crucial exponents that characterize the two-dimensional Ising model's universality class. According to finite-size scaling theory, power laws in relation to the system size L determine the properties of observables around the critical point [16].

An independent calculation of the correlation length exponent was made possible by the shift of the pseudocritical temperatures with system size. We found $\nu \approx 1.0$ by fitting the connection in Eq. 8, which fully agrees with the exact solution [3, 33] (see Fig. 3a). Once the correlation length exponent ν has been established, we can compute β by using a double-logarithmic scale to plot the magnetization at the critical point against the system size and fitting a linear regression (according to Eq. 9). The slope reveals $-\beta/\nu$. Applying these techniques to our Monte Carlo data yields $\beta = 0.122(3)$, which is in close agreement with the precise value of $1/8 = 0.125$ for the 2D Ising model [3, 33]. This confirms the correctness of our finite-size scaling approach, as Fig. 3b illustrates.

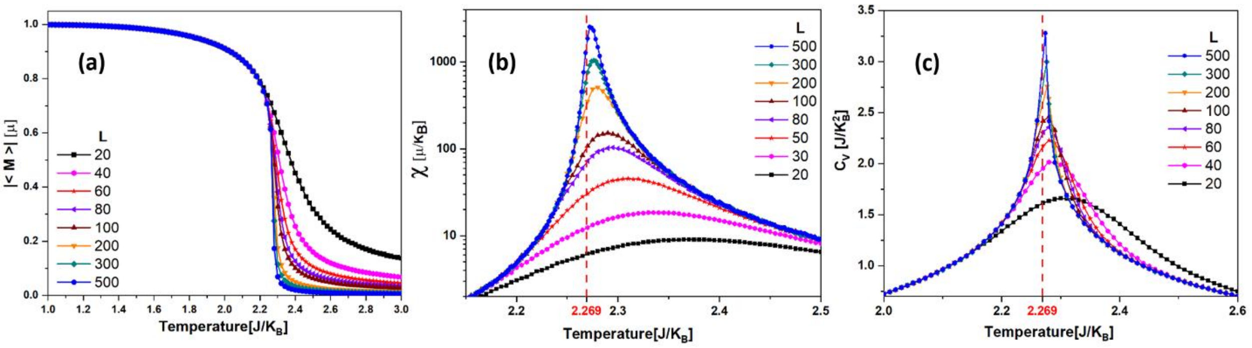


Fig. 1 *Finite-size effects in the two-dimensional Ising model: Temperature dependence of (a) magnetization M, (b) susceptibility χ, and (c) specific heat C_v for different lattice sizes L.*

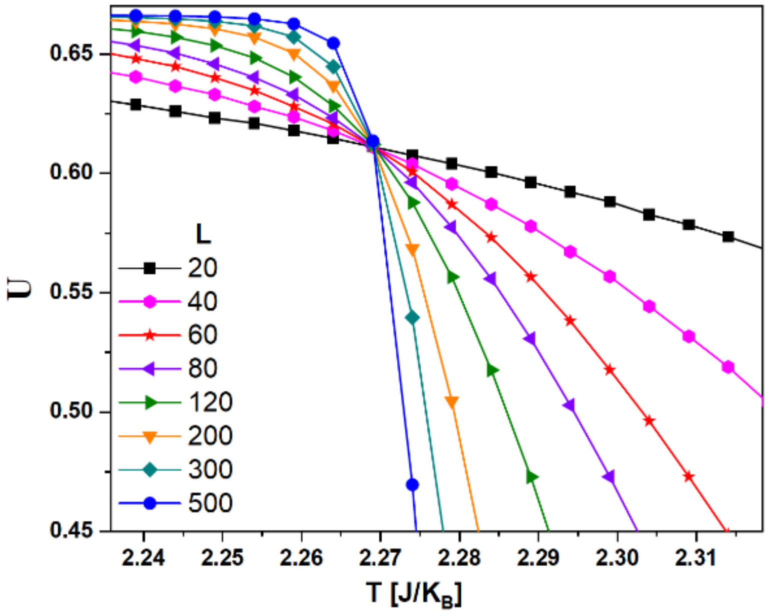


Fig. 2 *Binder cumulant U versus temperature for various system sizes L. The crossing point of the curves yields a size-independent estimate of the critical temperature* $T_C = 2.2690$.

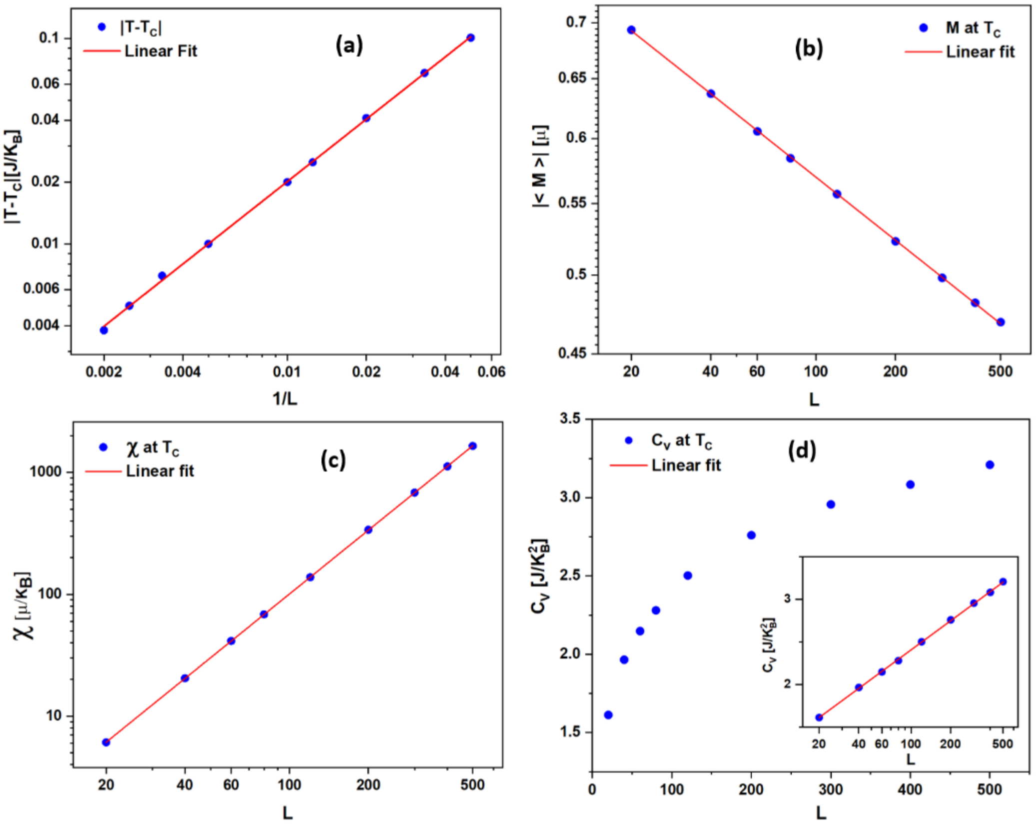


Fig. 3 *Finite-size scaling analysis of the critical exponents: (a) Correlation length exponent ν, (b) magnetization exponent β, (c) susceptibility exponent γ, and (d) specific heat exponent α. Main panel: Power-law scaling of the specific heat. Inset: Semi-logarithmic scaling*

A finite-size scaling analysis of susceptibility at the critical temperature can also be used to calculate the susceptibility critical exponent γ. A log–log plot of χ against L yields the slope γ/ν, allowing the determination of γ, as shown in Fig. 3c. At T_C, the susceptibility exhibits a scaling pattern with system size as given in Eq. 10. Using this method on our data, we obtain $\gamma = 1.74(1)$, which is astonishingly close to the exact value of $7/4 = 1.75$ for the 2D Ising model [3, 33]. The specific-heat exponent α indicates how the heat capacity behaves as it approaches T_C. In finite-size scaling, the peak value at T_C behaves like $C_v\,(T_C, L) \sim L^{\alpha/\nu}$ when $\alpha > 0$ [16, 25]. For the 2D Ising universality class, $\alpha = 0$, which means the divergence occurs logarithmically instead of a power law; thus, one anticipates this in Eq. 11 as shown in Fig. 3d. Hyperscaling connects exponents through the relationship $\alpha = 2 - d\nu$ [3], and with $d = 2$ and $\nu = 1$, this results in $\alpha = 0$, which aligns with the logarithmic behaviour [3, 33].

Disordered Ising model

The Monte Carlo simulation results for the two-dimensional Ising model under binary quenched bond disorder are presented and analyzed for lattice sizes $L = 20 - 300$. The primary goal is to investigate the impact of varying bond randomness strength on the system's fundamental characteristics and thermodynamic behavior. Three particular ratios of weak-to-strong exchange couplings were examined for the disordered systems: $r = J_2/J_1 = 0.25,\ 0.50,\ 0.75,$ corresponding to the bond pairs $(J_1, J_2) = (1.00, 0.25)$, $(1.00, 0.50)$, and $(1.00, 0.75)$ respectively. Each nearest-neighbor interaction was assigned J_1 or J_2 with equal probability according to Eq. 6 since the strong- and weak-bond probabilities were fixed at $p = 0.5$ for all situations. The temperature dependence of the Binder cumulant

$U(T, L)$ for the disordered two-dimensional Ising model is shown in Figure 4 for different lattice sizes L and disorder strengths r.

Fig. 4a displays strong intersections at $Tc \approx 2.12J/kB$ for weak disorder ($r = 0.75$), which closely resembles the behavior of the pure Ising model. This implies that, with only slight disorder-related disturbances, the system maintains the universal characteristics of the clean model. The intersection moves downward to $Tc \approx 1.955J/kB$ as the disorder strength rises to $r = 0.50$ (Fig. 4b), and the slope close to the crossover becomes less noticeable. This suggests that finite-size effects are enhanced and the transition is smoothed by disorder. The threshold temperature further drops to $Tc \approx 1.78J/kB$ for strong disorder ($r = 0.25$) in Fig. 4c, and the curves show wider crossings, indicating the progressive weakening of ferromagnetic correlations. Random-bond disorder successfully lowers the average coupling strength between spins, as shown by the lowering of Tc with decreasing r. Furthermore, it appears that the transition is still second-order based on the persistence of clearly defined crossings across all r values. According to the Harris criterion, however, disorder-induced changes in the critical exponents or universality class may be indicated by changes in the form and value of the Binder cumulant at criticality. These findings verify that disorder considerably modifies the thermal behavior and scaling features close to criticality, but it does not eliminate the ferromagnetic phase transition.

As a function of $1/L$ for various disorder concentrations r, Figure 5 displays the finite-size scaling analysis of the temperature variation $|T - Tc|$. The results, including logarithmic corrections, are shown in the insets. The simulation data were fitted using both polynomial and linear forms. Multiplicative logarithmic corrections in the scaling behavior are confirmed by the enhanced linearity in the insets. With very slight variations, the scaling for the weakly disordered system ($r = 0.75$) exhibits pure Ising behavior, suggesting that disorder functions as a marginal perturbation. These discrepancies become more noticeable as the disorder rises ($r = 0.50$ and $r = 0.25$), and the scaling is well represented by Eq. 18.

According to the analysis, $\nu \approx 1$ is largely constant at all disorder concentrations, which is in line with the Harris criterion for two-dimensional marginal disorder. However, the finite-size dependence is largely explained by the logarithmic correction exponent.

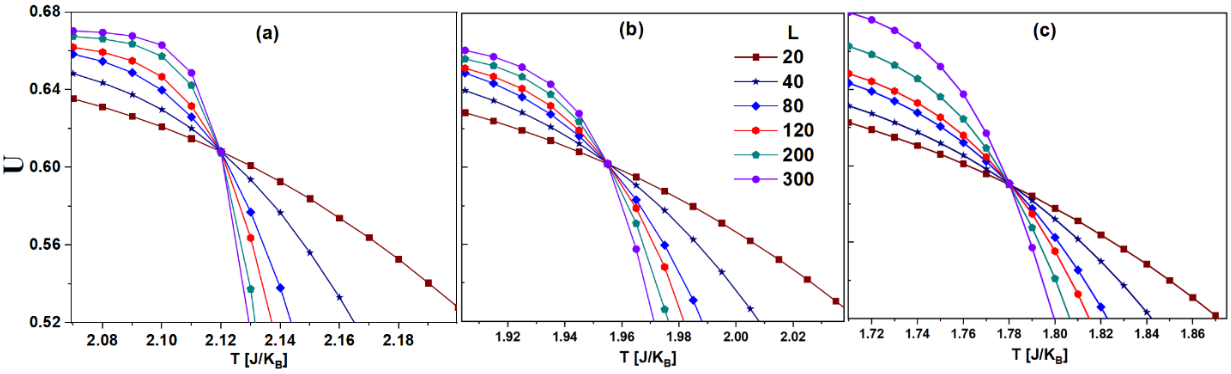


Fig. 4 *Binder cumulant U versus temperature T for the disordered Ising model at strong-bond concentration $p = 0.5$. Each panel shows data for a different disorder strength: (a) $r = 0.75$, (b) $r = 0.50$, and (c) $r = 0.25$.*

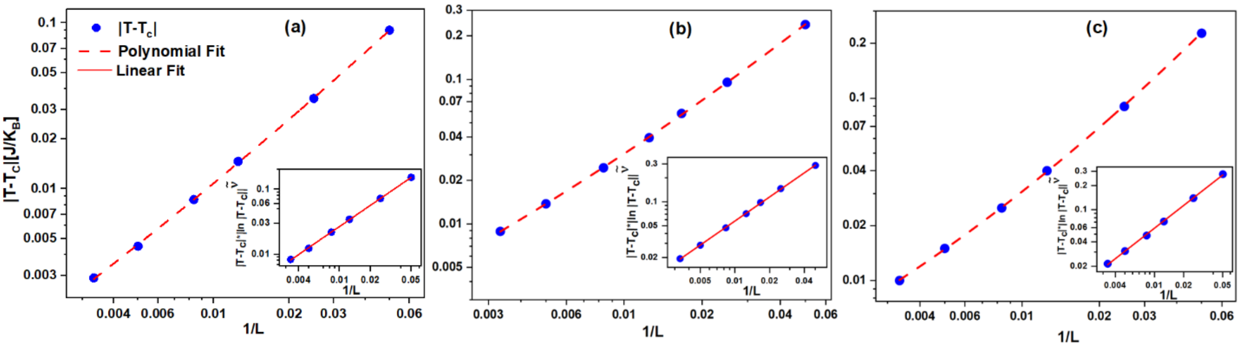
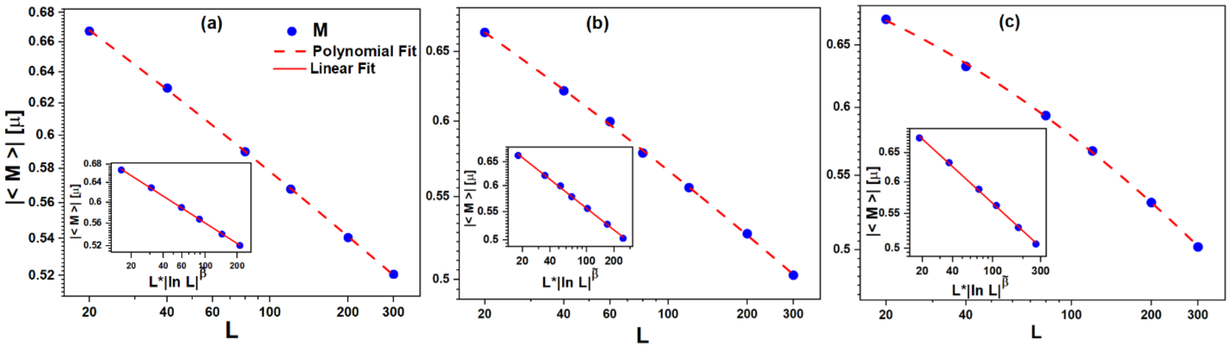


Fig. 5 *Finite-size scaling of the temperature deviation $| T - T_c |$versus $1/L$ for a different disorder strength: (a) $r = 0.75$, (b) $r = 0.50$, and (c) $r = 0.25$. Inset: Double logarithmic plot of $|T - T_C|^{\nu}| ln|T - T_C| |^{\tilde{\nu}}$ vs. $1/L$.*

In line with earlier theoretical and numerical research on the two-dimensional random-bond Ising model, a value of $\tilde{\nu} \approx 0.5$ offers a good explanation of the data for all r that have an excellent agreement with the theoretical one, $\tilde{\nu} \approx 0.5$ [7, 35]. Rather than a change in the leading critical exponent, the larger curvature seen for smaller r (stronger disorder) results from the increasing weight of the logarithmic factor.

For various disorder concentrations r in the quenched binary Ising model, Figure 6 displays the finite-size scaling of the magnetization M as a function of lattice size L. Both polynomial (dashed) and linear (solid) versions are used to match the simulation data, and logarithmic corrections are included in the insets to evaluate the updated scaling relation. The presence of multiplicative logarithmic corrections in the disordered regime is confirmed by the obvious improvement in linearity brought about by the addition of the logarithmic term.

The results are well represented by $\tilde{\beta} \approx 0.132$ for weak disorder (r=0.75; Fig. 6a), suggesting a slight correction to the pure scaling. The correction term grows greater with $\tilde{\beta} \approx 0.093$ at intermediate disorder (r=0.50; Fig. 6b), resulting in a more noticeable departure from the pure power law. The best fit is found with $\tilde{\beta} \approx 0.064$ in the extremely disordered scenario (r=0.25; Fig. 6c), which is in line with the theoretical value $\tilde{\beta} \approx 0.0625$ [7, 35]. While the leading Ising exponent β/ν largely stays constant, the increasing influence of disorder on the magnetization scaling is indicated by the steady decrease of $\tilde{\beta}$ with decreasing r. Although the quenched binary Ising model shows marginal logarithmic corrections whose strength varies on the disorder concentration, this behavior verifies that the model stays in the Ising universality



class. The scaling form with the logarithmic term is further supported by the outstanding data collapse in the insets.

Fig. 6 *Finite-size scaling of the magnetization* $|\langle M\rangle|$*versus lattice size* L*for disorder concentrations: (a)* $r = 0.75$*, (b)* $r = 0.50$*, and (c)* $r = 0.25$*. The data follow the relation in Eq. 15. Inset: Shows the improved linearity after including the logarithmic term* $|\ln L|^{-\tilde{\beta}}$*.*

Figure 7 shows the finite-size scaling of the susceptibility χ with system size L for various disorder concentrations r. The power-law fits (Eq. 16) are displayed in the main panels, and the updated scaling form is tested using logarithmic adjustments in the insets. The presence of multiplicative logarithmic corrections in the disordered regime is confirmed by the substantial improvement in linearity brought about by the addition of the logarithmic factor.

The findings agree well with $\tilde{\gamma} \approx 0.68$ for weak disorder (r=0.75; Fig. 7a), suggesting a slight correction to the pure Ising scaling. The correction exponent grows to $\tilde{\gamma} \approx 0.765$ as the disorder develops ($r = 0.50$; Fig. 7b), indicating stronger deviations from the clean limit. $\tilde{\gamma}$ climbs to about 0.87 in the extremely disordered scenario ($r = 0.25$; Fig. 7c), which is in close agreement with the theoretical and numerical estimate $\tilde{\gamma} \approx 0.875$ published in previous investigations of the random-bond Ising model [7, 35].

The gradual intensification of disorder-induced fluctuations in the magnetization is reflected in this monotonic increase of $\tilde{\gamma}$ with decreasing r, whereas the leading Ising exponent stays almost constant. As a result, the quenched binary Ising system maintains the Ising universality class while displaying logarithmic corrections that are dependent on disorder and whose size increases with disorder strength. The scaling form in Eq. 20 is further supported by the outstanding linear behavior in the insets over the whole range of r.

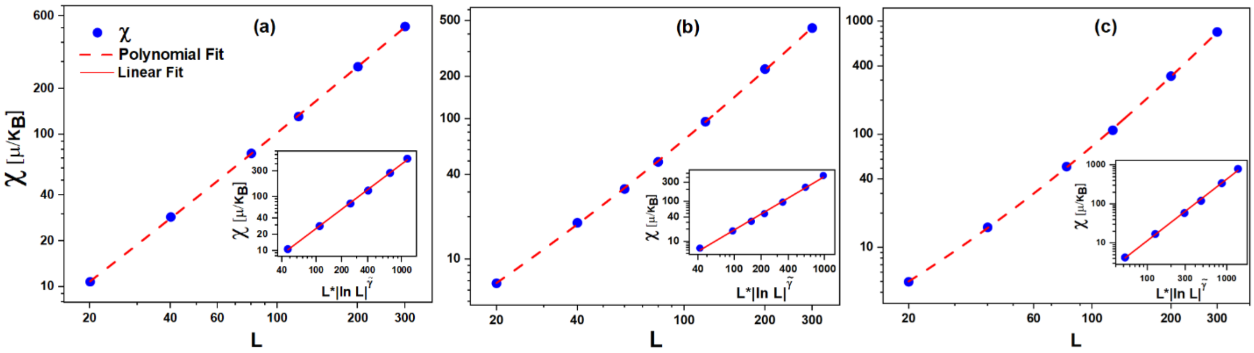


Fig. 7 *Finite-size scaling of the susceptibility* χ *versus lattice size* L *for disorder concentrations: (a)* $r = 0.75$*, (b)* $r = 0.50$*, and (c)* $r = 0.25$*. Insets: The data follow the form* $\chi \sim L^{\gamma/\nu}(\ln\ L)^{\tilde{\gamma}}$ *with logarithmic correction exponents* $\tilde{\gamma} \approx 0.68, 0.765,$ *and* 0.87 *as* r *decreases.*

Conclusions

We have performed a detailed finite-size scaling analysis of the disordered quenched binary Ising model to examine the effect of quenched randomness on the critical behavior of the clean two-dimensional Ising universality class. When disorder was introduced through a binary distribution of exchange contacts, systematic alterations were seen in the logarithmic correction terms that accompanied the scaling relations of the correlation length, magnetization, and susceptibility. The results demonstrate that disorder acts as a marginally irrelevant disturbance in two dimensions, whereas the leading critical exponents are consistent with the pure Ising universality class. Although the logarithmic correction exponents obviously depend on the disorder concentration, stronger disorder produces more pronounced corrections. The correction exponents specifically approach values reported in previous theoretical research, while the Ising universality class remains under quenched disorder. These findings corroborate the theoretical prediction that quenched randomness modifies the scaling behavior in a quantifiable logarithmic way without altering the 2D Ising model's universality class. Thus, our work provides more numerical evidence for the marginal impact of disorder in two-dimensional systems and advances our knowledge of crucial occurrences in disordered magnetic models.

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دراسة مونت كارلو للتحولات الطورية في نموذج إيزينج في ظل اضطراب الرابطة الثنائية المُخمَّدة

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الخلاصة:

تم دراسة تأثير الاضطراب المُخمَّد على السلوك الحرج لنموذج إيزينج ثنائي الأبعاد دراسةً منهجيةً باستخدام محاكاة مونت كارلو واسعة النطاق وتحليل التدرج ذي الحجم المحدود. وللتحقق من الأسس الحرجة المميزة لفئة العالمية الصرفة، فُحص نظام إيزينج ثنائي الأبعاد النظيف أوليًا. ثم استُخدم توزيع ثنائي لارتباطات التبادل بتركيزات اضطراب مختلفة لإدخال الاضطراب، مما أتاح تحليلًا شاملًا لتأثيراته على خصائص التدرج بالقرب من درجة الحرارة الحرجة. حُللت المعلمات الديناميكية الحرارية مثل طول الارتباط، والمغنطة، والقابلية، وأظهرت النتائج أن الاضطراب يُسهم في تصحيحات لوغاريتمية كبيرة لقوانين التدرج، بينما تبقى الأسس الحرجة الرئيسية قريبة من تلك الخاصة بنموذج إيزينج النظيف. تُثبت هذه النتائج أنه على الرغم من التغييرات اللوغاريتمية المعتمدة على الاضطراب في سلوك التدرج، فإن نموذج إيزينج الثنائي المُخمَّد يحافظ على فئة عالمية إيزينج. إن المزيد من الأدلة على أن الاضطراب الضعيف في بعدين يسبب اضطرابات غير ذات صلة إلى حد ما والتي توفر تصحيحات لوغاريتمية للمقياس الحرج يتم توفيرها من خلال النتائج الحالية، والتي تتفق مع التوقعات النظرية من تحليل مجموعة إعادة التطبيع والتحقيقات العددية السابقة.