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Co-retractable Modules and Semi-Simplicity Property

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Abstract

Co-retractable modules appeared in 2006 by Clark, Lomp and Vanaja. In this article, we present and study the relationships between two known concepts namely semi-simple modules and Co-retractable module. Our concern in this article is to develop some properties of Co-retractable module and related to other classes of modules such as multiplication module, indecomposable module and projective module. We prove that if M is a multiplication R-module with $\mathrm{ann}_R(M)$ is a prime ideal and the Socal of M is a direct summand of M, this means M is a retractable module. Also, if M is an indecomposable multiplication R-module with $\mathrm{ann}_R(M)$ is a prime ideal of Noetherian Ring R and every direct sum of modules with summand intersection property has summand sum property, then M is a co-retractable module. Finally, if M is a simple quasi-Dedekind module over the Artinian Ring R with rad(R) = 0 and $\mathrm{ann}_R(M)$ is a prime ideal of R, then M is a co-retractable module.

Introduction

All rings are commutative with unity and all modules over the ring R is a unitary. Any ring R with unity has no zero divisors is called division if for all a in R is invertible. From [1] a ring R is called simple if $R \neq 0$ and for every submodule A of R as R-module, A = 0 or A = R. A module M is called multiplication if each A submodule of M and I is an ideal of R, so $A = IM$. An ideal I of the ring R is called prime ideal if $\forall$ a ,b $\in$ R such that a.b $\in$ I, then either a $\in$ I or b $\in$ I [2]. M is said to be indecomposable if whenever $M = M_1 \oplus M_2$, then $M_1 = (0)$ or $M_2 = (0)$ [3]. Note that every division ring is simple and also, we can say if R is a simple this means a multiplication R-module is simple and hence is Co-retractable module [4]. A module M is called Co-retractable if we have A is proper submodule of M and $f:M/A \longrightarrow M$, so f is nonzero [4]. A module M is said to be retractable in case for every $0 \neq A$ is a submodule of M, so, $\mathrm{Hom}_R(M,A)$ is a nonzero [5]. By other words, M is Co-retractable if $0 \neq f \in \mathrm{End}(M)$ such that $f(A) = 0$, where A is a proper submodule of M. A module M is called simple if it has only two submodules are {0} and itself M, so M is called semi-simple if it is a direct sum of simple module [6].

Any module Mover integral domain is said to be divisible if rM = M, r $\in$ R. A module M is called Quasi-Dedekind module (Q-Dedekind module) [7]. If $Z(M) = \{x \in M \ni \mathrm{ann}_R(x)$ is essential ideal of R$\}$, so Z(M) is called singular submodule in M [8]. Also, if Z(M) = M, this means M is singular submodule and when Z(M) = 0, then M is called nonsingular module. Ghorbani and Vedadi [9] studied Epi-Retractable module and some application. Also, Lee [10] studied multiplication module with endomorphism rings which are integral domain and also, he found some important notions about the topic. A proper submodule A of M is called essential if the intersection of A with any nonzero submodule B of M is a proper submodule of M [11]. Any module M is said to be uniform if every submodule of M is essential. Recall that a submodule L of M is called fully invariant provided $\phi(L) \subseteq L$ for every endomorphism ϕ of M [12]. Let A be a finite left R-module. We will say that A satisfies the sole condition if, for any $a \in \text{soc}(A)$, any monomorphism $f: Ra \to A$ can be extended to A [13].

A commutative ring R is called Noetherian if each ideal in R is finitely generated [14]. An R -module M is called quasi-Dedekind if Hom(M / N , M) = 0 for all nonzero submodules N of M for all nonzero submodules N of M[15]. R is called Von Neumann regular ring if for all x ∈ R, there exists y belongs to R such that xyx = x[16]. An R-module M is called Artinian if M satisfies the descending chain condition (DCC) on submodule of M[17]. An R-module M is called injective if for every monomorphism h: $M_1 \to M_2$ and homomorphism f: $M_1 \to M_3$ there exists a homomorphism g: $M_2 \to M_3 \ni g \circ h = f$ [18]. A module P over a ring R is said to be projective if given any diagram of R-module homomorphisms [19]. A module M is weakly injective if for every finitely generated submodule N of the injective hull E(M) of M there exists $X \subseteq E(M)$ such that $N \subseteq X \cong M$ [20].

In this paper, we study Co-retractable module and related to simple and semi-simple concept. Finally, some new results with several properties are presented.

Results

In this section, we introduce and study two concepts namely simple and semi-simple modules and related to Co-retractable modules. Some other concepts have been studied with Co-retractable module. Also, some definitions and properties have been introduced. A module M is called Co-retractable if $0 \neq g: M/A \longrightarrow M$ where A is a proper submodule of M and g is a homomorphism.

Definition 2.1.[21]. Let M be an R-module. If the following conditions are hold, then M is called semi-simple module:

(1) A is a submodule of M such that $A = \sum K_i$, i=1,…,n are simple submodule.
(2) $M= \sum A_i$, i = 1,…, n are simple submodules.
(3) $M = \oplus \sum A_i$, i = 1,…,n
(4) $M = N \oplus K$.

Remarks and Example 2.2.

1. The {0} is a semi-simple module such that $\{0\} = \sum A_i$, i = 1,…, n, are semi-simples. Not that {0} is not simple module.
2. The ring R is semi-simple if every R-module M over the ring R is semi-simple module. So, M is Co-retractable module.
3. For $0 \neq g \in \text{End}_R(M)$ and $A \leq M$, so $A \subseteq \text{Ker}(g)$.
4. Every semi-simple module M is a Co-retractable module.
5. As an example of not Co-retractable module, we present the rational numbers Q and the integer numbers Z.

Lemma 2.3. [13] Let M be a multiplication R-module with $ann_R(M)$ is a prime ideal of R. If M is a semi-simple, then it is a Co-retractable module.

Proposition 2.4. Let M be a multiplication R-module with $ann_R(M)$ is a prime ideal. If the Socal of M is a direct summand of M, then M is a retractable module.

Proof: We know $Soc(M)=\oplus M$. To prove that $K= 0$. Suppose that $K\neq 0$. Let $0\neq k \in K$ and let $J= \{r\in R, r K=0\} = ann_R(K)$. Hence J is an ideal of R and $R/J \simeq R_R$. But J is a proper ideal of R, so I is a maximal ideal of R ∋ R/ I is simple R-module. But $R/ I \simeq R_K/ I_K$. So R_K/ I_K is a simple R-module. Since $I_k \leq M$, then I_k is a direct summand of M. Since $I_k \leq R_K \leq M$, then I_k is a direct sum of R_k. Hence $R_k = I_k \oplus L$. So, $L \leq R_k / I_k$. So is a simple module. Therefore, $L \leq Soc(M) \leq M$. Then $Soc(M) \cap L = L \leq R_k \leq K$ and then $K \cap Soc(M) \neq \phi$ but this contradiction. Hence $K =0$ and then $M = Soc(M)$. Therefore, M is a semi-simple module. Thus, M is a retractable module.

Proposition 2.5. Let N be a submodule of a multiplication R-module with $ann_R(M)$ is a prime ideal an R-module M. If N is the intersection of essential submodule of M, then M is a semi-simple module and hence is a Co-retractable module.

Proof: Suppose that $Soc(M) \subseteq N$. Let $K = \cap\{L: L \leq_{ess} M \text{ and } N \leq L\}$. So $N \leq M$, we have that $N = K$. Let $J \leq K$ and $N \leq J$. We claim that J is a direct summand of K. Let B be a direct of J in M ($N \leq J \leq K$). So $\oplus B \leq_{ess} M$. Since $N \leq J\oplus B$, then $K \leq J\oplus B$ and hence $J \leq K \leq J\oplus B$. Hence J is a direct summand of K. Since $N\leq N$ and $N\leq K$, then N is a direct summand of K. Hence $K=N\oplus T$ ∋ $T \leq K$. So $N \leq N\oplus F$. Hence $N\oplus F$ is a direct summand of K. Therefore, F is a direct summand of K. Now $F \leq T \leq K$, so F is a direct summand of T. So, T is a semi-simple module (M is a Co-retractable module).

We can defind a free module as If F has a nonempty basis X, then F is a free R-module on the set X.

Corollary 2.6. Let R be a semi-simple ring with $ann_R(M)$ is a prime ideal. Then every R-module M is a Co-retractable module.

Proof: We must prove that M is a semi-simple module over the ring R. There is a free presentation.

$$0\longrightarrow N\longrightarrow F\longrightarrow M\longrightarrow 0$$

So, M is a quotient of free module F over R. Clear F is copy of the ring R and R is a semi-simple ring, this means F is a semi-simple. Hence, M is also semi-simple module. Thus, M is a Co-retractable module.

Corollary 2.7. Let M be a ring with $ann_R(M)$ is a prime ideal. If M is a factor of projective R-module, then M is a Co-retractable module.

Proof: Let M be an R-module. We a short exact sequence:

$$0\longrightarrow M\longrightarrow R\longrightarrow R/M\longrightarrow 0$$

Since any module of this ring is projective, then R/M is also projective. Hence this sequence is split. So, $R \cong M\oplus M$ ∋ $N \subseteq R$ and N is submodule and isomorphic to R/M. Hence R is semi-simple. So, M is semi-simple module, with $\text{ann}_R(M)$ is a prime ideal, M is a Co-retractable module.

Recall that if $R/\text{ann}(M) = R^*$, so any R^*-module is Co-retractable. Also, if $M_1,\ldots,M_n$ are Co-retractable, this imply that $M_1 \oplus \ldots \oplus M_n$ is also Co-retractable.

Definition 2.8.[11]. The radical of M, written as rad(M), is the intersection of all maximal submodules of M. When M=R, this is also called the Jacobson radical and denoted J(R).

Definition 2.9. [22]. A submodule N of M is called a small submodule of M if whenever N+ K= M for some submodule K of M, we have M = K, and in this case we write N << M.

Proposition 2.10. Let M be indecomposable multiplication R-module with $\text{ann}_R(M)$ is a prime ideal. If $N \leq M$ is addition complement, then M is a Co-retractable module.

Proof: If the radical does not equal zero, so $\exists\ 0 \neq r \in \text{rad}(M)$, then Rr is addition complement ($Rr + L = M$); $Rr \cap L$ is a small in $Rr \ni L \leq M$. Since $r \in \text{rad}(M)$ and Rr is a small in M, then Rr is a small in Rr ($L = M$) and this contradiction. ($\text{rad}(M) = 0$). Suppose that B is addition complement submodule of M such that $B+B_1 = M$ and $B \cap B_1 \ll B$; B_1 subset of $(B \cap B_1) \oplus \text{rad}(M) = 0$; $B \cap B_1 = 0$. Then $B \oplus B_1 = M$ and this means M is a semi-simple module. But M is a multiplication module and $\text{ann}_R(M)$ is a prime ideal (M is a Co-retractable module).

Proposition 2.11. Let A be a submodule of projective R-module M and $A=IM$. If R is a hereditary ring with $\text{ann}_R(M)$ is a prime ideal and End(M) is local ring, then M is a Co-retractable module.

Proof: Since M is a projective module, then it is injective module. We have R is a hereditary ring. Then M has summand sum property. Let

$A \leq R$. Let F be a free module and $\varphi: F \longrightarrow A$. So, $F \oplus R$ is a projective module and has summand sum property. Let $\psi: A \longrightarrow R$ and $\lambda: F \longrightarrow R \ni \lambda = \varphi\psi$. So $I\text{mg}(\lambda) = A$ is a direct summand of R. Hence R is a semi-simple ring (M is semi-simple module). But M is multiplication module with $\text{ann}_R(M)$ is a prime ideal imply that, M is a Co-retractable module.

Proposition 2.12. Let M be a perfect and projective multiplication R-module with $\text{ann}_R(M)$ is a prime ideal. If every weakly injective module is injective; then M is a Co-retractable module.

Proposition2.13. Let M be an indecomposable multiplication R-module with $\text{ann}_R(M)$ is a prime ideal of Noetherian ring R. If every direct sum of modules with summand intersection property has summand sum property, then M is a Co-retractable module.

Proof: Suppose that M is injective module. We have R is a Noetherian ring. Then M is a direct sum of indecomposable modules. But indecomposable modules have summand intersection property. So, R is a semi-simple ring. Hence M is a semi-simple R-module. We have M is a multiplication module with $\text{ann}_R(M)$ is a prime ideal. (M is a Co-retractable module).

Corollary 2.14. Every semi-simple R-module is Co-retractable module.

Corollary 2.15. Every module over semi-simple ring is Co-retractable module.

Lemma 2.16: [13] Let M be a quasi-Dedekind simple R-module. Then M is Co-retractable module.

Proposition 2.17. Let M be a simple quasi-Dedekind module over the ring R. If R is Artinian ring with rad(R) = 0 and $ann_R(M)$ is a prime ideal of R, then M is Co-retractable module.

Proof: If R is Artinian ring; then we have $\cap_i^n X_i$ = rad (R) = 0 such that X is maximal ideal of R. Take the mapping from R into $(R/x_1 \oplus \ldots \oplus R/x_n)$. So $(R/x_1 \oplus \ldots \oplus R/x_n)$ is a semi-simple. Hence R is also semi-simple (M is a semi-simple module with $ann_R(M)$ is a prime ideal imply that M is a Co-retractable module).

Not that by theorem due to Kaplansky a commutative ring R is von Neumann regular if and only if every simple R module is injective. Therefore, the next theorem explains the relationship between regular ring and Co-retractable module.

Theorem 2.18. Let M be a multiplication R-module with $ann_R(M)$ is a prime ideal. If every simple R-module M is injective, then M is a Co-retractable module.

Proof: Let M be a ring R-module over the ring R such that every simple R-module satisfy injective. If $0 \neq m \in M$, so $A \leq M$ which is maximal among the submodules H of M and $m \notin H$ (Zorn's lemma). Let T = ∩ {all submodules B of M and $B>A$}. Then $m \in T$, and $T/A \neq 0$ is simple. Therefore $M/A = (T/A \oplus H/A)$, where H is a submodule of M. Since x cannot be contained in H, it follows that A is a maximal submodule of M. Hence M is semi-simple, because $m \notin A$. But M is a multiplication module, with $ann_R(M)$ is a prime ideal imply M is Co-retractable module.

Lemma 2.19. Let M be an R-module that is a sum of simple submodules $\{M_i\}_{i \in I}$, and let N be an arbitrary submodule of M. Then there is a subset $J \subseteq I$ such that $M \cong N \oplus (\oplus M_i, i \in J)$.

Theorem 2.20. Let M be a multiplication module. If R is a semi-simple ring with $ann_R(M)$ is a prime ideal, then M is Co-retractable module.

Proof: Let M be an R-module. Then M has a free exact sequence

$$0 \longrightarrow K \longrightarrow F \longrightarrow M \longrightarrow 0.$$

So M is a quotient of F. Since F is a direct sum of copies of R and R is assumed to be semi-simple, so F is semi-simple. We have M is a multiplication module and $ann_R(M)$ is a prime ideal. So, M is Co-retractable module.

Recall that any module M is called simple if it has only two submodules are {0} and itself M.

Example 2.21. Z_P is Co-retractable module where p is prime number, because Z_p is simple module.

Given an R-module M and submodules L, N of M then $(N:_R L)$ will denote the set of elements $r \in R$ such that $rL \subseteq N$. Note that $(N:_R L)$ is an ideal of R for all submodules L, N of M. We now define the R-module M to be a fully invariant multiplication module in case for each fully invariant submodule K of M there exists an ideal G of R such that K = GM. It is clear that the module M is a fully invariant multiplication module if and only if $K = (K:_R M)M$ for every fully invariant submodule K of M. Clearly a module M is a multiplication module if and only if M is a fully invariant multiplication module and every submodule of M is fully invariant. It is also clear that any isomorphic copy of a fully invariant multiplication module is also a fully invariant multiplication module.

Definition 2.22. [16]. Let M be a module. Then M is called torsion free if its torsion submodul

Lemma 2.23. [19] Every faithful multiplication module over division ring $\mathrm{End}_R(M)$ is simple module and hence is Co-retractable module.

1) Let R be any ring and let M be any fully invariant multiplication module over the ring R. Then the R-module $M(I)$ is a fully invariant multiplication module for every index set I.

2) Let R be a domain and let M be a non-zero torsion-free R-module. If L is a fully invariant submodule of M then $aL \subseteq bL$ for all elements a,b in R such that $aM \subseteq bM$.Moreover, the converse holds if M is uniform.

Proposition 2.24: Let R be a domain. Then every torsion-free divisible module over division ring $\mathrm{End}_R(M)$ is Co-retractable module.

Proof: Let F denote the field of fractions of R. Let M be any non-zero torsion-free divisible R-module. It is well known that M is a vector space over F and hence the R-module $M = \sim F(I)$ for some index set I. By (1) it is sufficient to prove that the R-module F is a fully invariant multiplication module and thus we can suppose without loss of generality that M is uniform. Let L be a non-zero fully invariant submodule of M. For each non-zero element $a \in R$, $M = aM$ and hence $L = aL$ by (2). Thus $L = aL$ for each $0 \neq a \in R$, so that L is divisible, hence injective and a direct summand of the uniform module M. We conclude that L = M = RM. Thus, M is a fully invariant multiplication module. So, M is simple module and hence is Co-retractable module, as required.

Lemma 2.25. [19] Let M be a faithful multiplication module. If every proper submodule A of M is prime, then M is simple module.

(3) Let R be any ring and let M be any multiplication module over the ring R. Then the R-module $M(I)$ is a fully invariant multiplication module for every index set I.

Proof: By (1).

(3) Let R be any ring. Then every free R-module is a fully invariant multiplication R-module.

Proof. By (3) because the R-module R is a multiplication module.

Theorem 2.26. Let R be a Dedekind domain. If A is a prime submodule of finitely generated torsion-free R-module M, then M is simple module and hence Co-retractable module.

Proof: Let M be any finitely generated torsion-free R-module. If $M \cong A$ for some non-zero ideal A of R then M is a multiplication module. On the other hand, if M is a free R-module than M is a fully invariant multiplication module by (3). Thus, M is simple module and so is Co-retractable module.

Proposition 2.27. Let M be a divisible module over integral domain R. If JM = A where J is an ideal of R and A is a proper submodule of M, then M is a Co-retractable module.

Proof: Suppose that JM = A and $A \leq M$. Then there exists J is an ideal of R. So, M is a multiplication module. But M is divisible module. Hence A = M. Therefore, M is a simple module. Since every simple module is a semi-simple module. Thus, M is Co-retractable module.

Corollary 2.28. L et R be a simple ring. Then every homomorphic image of multiplication module is a Co-retractable module.

Proof: Assume that M is a multiplication module, $d: M \rightarrow M^*$ is R-homomorphism and H = d(M). Suppose that h belongs to H. So, h = d(a), $a \in M$. By assumption and (1), there exists an ideal J of R such that Ra = JM. So

$$\begin{aligned} Rh &= R(d(a)) \\ &= d(Ra) \\ &= d(JM) \\ &= JH. \end{aligned}$$

Therefore, H is a multiplication module. But R is simple ring. Then H is simple module and hence Co-retractable module.

Proposition 2.29. Let M be a faithful multiplication over the ring R. If M is a semi-simple module over semi-simple ring $S = End_R(M)$, then is a semi-simple over the ring R and hence M is Co-retractable.

Proof: Assume that M is semi-simple over S. But, M is a multiplication module over R. From [Theorem 2.43 in [19], M is a faithful module. Since $End_R(M)$ is semi-simple ring, then S is a semi-simple ring. From M is a faithful multiplication module and [Theorem 2.44 in[19], then M is a finitely generated multiplication module over R. Therefore, by [Theorem 2.43 in [19], $S \cong R$. Since S is a semi-simple ring, then R is a semi-simple ring. Hence M is a semi-simple R-module. Thus, M is a Co-retractable module.

Corollary 2.30. Let M be a multiplication module over integral domain $S = End_R(M)$. If every $0 \neq g$ is onto homomorphism of S, then M is simple module.

Corollary 2.31. Let M be a multiplication module over simple ring R. Then M is simple module.

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المقاسات القابلة للسحب وخاصية شبه بسيطة

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الخلاصة:

تمت دراسة العلاقات بين مفهومين معروفين هما المقاسات شبه البسيطة والمقاسات القابلة للسحب. ينصب اهتمامنا في هذا البحث على تطوير بعض خصائص المقاسات القابلة للسحب والمتعلقة بفئات أخرى من المقاسات مثل المقاس المتعدد والمقاسات غير القابلة للتحلل والمقاس الإسقاطي. لقد أثبتنا أنه إذا كانت M مقاس متعدد و(annR(M)) هو مثالي اولي مع (soc(M))، فهذا يعني ان Mهي مقاس مسحوب. أيضا أثبتنا ان إذا كانت M هي مقاس متعدد غير قابل للتحليل مع (annR(M)) هو مثالي اولي مع كل المقاسات التي تحقق خاصية الجمع المباشر فهذا يعني انM هي مقاس قابل للسحب. أخيرا إذا كانت M هي مقاس بسيط على الحلقة الإريترية R مع (rad(R) = 0) و (annR(M)) هو مثالي اولي على R فان M مقاس قابل للسحب.