IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

SJPAS

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

Projective Tensor of Almost Kahler Manifold

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https://doi.org/10.54153/sjpas.2024.v6i1.799

Article Information
Received: 25/01/2024
Revised: 24/02/2024
Accepted: 28/02/2024
Published: 30/03/2024

Keywords:
projective tensor, Almost kahler manifold, Almost Hermitian Manifold.

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Abstract

The Work from this search is analyze the geometrical characteristics of conharmonic tensor of projectine Almost kahler menifold. we use the projective curvature tensor's flatness properties to determine the projective tensor's Almost kahler manifold compounds. (AK-manifold). And found some components Projective of Almost Kahler**.** proved that the manifold M is a flat holomorphic sectional tensor of projective Almost Kahlar manifold. Prove that the manifold M has J-invariant Ricci tensor. Proved that (AK-manifold) M is kahler manifold. Finally, There exists relation between Almost Kahlar manifold (AK-manifold) and Lacally confermal Kähler menifold (LCK-manifeld) has been found.

Introduction:

The symbol for one among the most popular significant classes of nearly structures is $W_1 \oplus W_4$, where $W_1$and W_4 and similarly indicated by the naerly Kähler menifold ánd conformal locälly Kähler manifóld. Gray and Hervella[4] proved that $W_1 \cup W_4 \subset W_1 \oplus W_4$ many properties of this subclass of the almost Hermitian structure linked closely with the properties of neärly Kóhler and lócally confórmal Kähler manifolds. On the other hand, the manifold $W_1 \oplus W_4$ does not coincide with $W_1$and W_4.

Among the most significant topics in differential geometry to create the differential geometrical structure synthesis is the almost Hermitian manifold (AH-manifold). In 1960, Koto [14] gave his first attempts and found a relation which was almost identical to the Kähler manifold and was considered as an entry to the manifold. In 1965 Gray[5], found an extreme method to build certain examples from AH-manifold, So the research continued until 1980, when Gray & Hervela released their most important results [6]. We have noticed most of the studies of this object were done by the language of invariant Koszul [12], but the object will be more suitable if it is studied by the method of adjoint G-structure by Kartan [9]. Kirichinko , who did a big change of this study ,when he found two new tensors, namely, structure and virtual tensors [11]. Elham Mawlood Mouamed [2] in 2021 studied N Generalized Conharmonic Curvature Tensor of the Locally Conformal Kahler Manifold. Ali Kalaf [1] in 2022, studied M-projective Curvature Tensor of Nearly Kahler Manifold. Finally, Yildirim &

Dirik [36] studied certain curvature tensors including the pseudo-projective on some contact metric manifolds.

In this paper study projective tensor of Almost kähler manifold (AK- manifold), & found some components Projective of Almost Kahler. & proved that the manifold M is a flat holomorphic sectional tensor of projective Almost Kahlar manifold & proved that the manifold M has J-invariant Ricci tensor. Proved that (AK-manifold) M is kahler manifold. Finally, There exists relation between Almost Kahlar manifold (AK-mánifold) and Lócally confórmal Kóhler manifold (LCK-manifold) has been found.

Preliminaries

X(M) should be the smóoth surface. M is avector field module. Let $C^{\infty}(M)$ represent an operations set on M. "A set $\{M, J, g = \langle .,. \rangle\}$," "where M is" a 2n-dimensional "(n>1) smoóth" manifold, is the manifold Hermitain (H M). $\hat{J}$ is an endomorphism of tengent space. $T_P(M)$, $(J_P)^2$=jd g=⟨.,.⟩ Matric Reimann "on M $\langle JZ, JW\rangle =$ "$\langle Z, W\rangle; Z, W \in Z(M)$"[13]. The besis "$\{e_1 \ldots, e_n \ldots Je_1 \ldots . Je_n\}$" is referred to as $\{J, g\}$; the new is constructed using this basis as follows "$\{i_1, \ldots i_n, \ldots \bar{i}_1, \ldots \bar{i}_n\}$"."Where $i_a=\sigma(e_a)$ and $i_a = \bar{\sigma}\,(e_a)$". The basis is referred to "almost structure". The farmer's corresponding"$\{P, \ldots i_1, \ldots, i_n \ldots, \bar{i}_1, \ldots, \bar{i}_n\}$" This an A-frame".

"The indicators u, g, l and P in the vicinity" "1,...,2n . The" We'll make use of the figures "1,2,...,k. Utilize" the indicators "$\{i_{\hat{1}} = \bar{i}_1, \ldots . i_{\hat{n}} = \bar{i}_n\}$" "where $\hat{a} = a + n$.than form can be used to write" a-frame"$\{p, i_1 \ldots, i_n \ldots, i_{\hat{1}}, \ldots i_{\hat{n}}\}$". The fallowing Q-struct forms are ad joint by the components matric of the complex structure ỷ and f:

$$"(\langle JX, JY\rangle J^i_j)" = "\begin{pmatrix} \sqrt{-1}\, I_n & 0 \\ 0 & -\sqrt{-1}\, I_n \end{pmatrix}", \quad "(g^i_j)" = "\begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}," \qquad (1)$$

"Where I_n is the rank n unit matrix"[11].

Definition2.1 [9]

Almost Hermitical structure (AH-structure) on M is a pair of tensor { J , g = < .,. > }, where g =<.,. > is a Riemannian metric and J is an almost complex structure, so that < JẊ, JỶ >=< Ẋ, Ỷ>, Ẋ, Ỷ ∈ Ẋ (M)

Theorem2.2[7]

The arrangement of the AK-manifold structure equations in the adjoined Q-structure takes the following forms:

1) $d\omega^a = \omega^a_b \wedge \omega^b + B^{abc}\omega_b \wedge \omega_c$;
2) $d\omega_a = -\omega^b_a \wedge \omega_b B_{abc}\omega^b \wedge \omega^c$;
3) $d\omega^a_b \omega^a_c \wedge \omega^c_b + B^{adc}_b \omega_c \wedge \omega_d + B^a_{bcd}\omega^c \wedge \omega^d + \left(A^{ac}_{bd} + 2B^{ach}B_{hbd}\right)\omega^d \wedge \omega_c$;
4) $dB^{abc} = B^{abc}_b \omega^d + B^{abcd}\omega_d - B^{dbc}\omega^a_d + B^{adc}\omega^b_d + B^{adb}\omega^c_d$;

Definition.2.3 [15]

A tensor of Riemannian R. for smooth menifold. M is four-covariant tensor $R: L_P(M) x L_P(M) x L_P(M) x L_P(M) \to \mathbb{R}$, as it characterized by

$$R[(S, T, U, V)] = (R(U, V)T, S)$$

$R(S, Y)U = ([\nabla s, \nabla t] - \nabla[S, T])U$,

That is $S, Y, U, V \in L_P(M)$, and meets each of follows criteria:

a) $R(S,T,U,V) = -R(T,S,U,V)$;
b) $R(S,T,U,V) = -R(S,T,V,U)$;
c) $R(S,T,U,V) = R(U,V,S,T)$;
d) $R(S,T,U,V) + R(S,U,V,T) + R(S,V,T,U) = 0$;

Theorem 2.4[7]

The following forms are given for the elements of the NK-Riemann manifold's curvature tensor in adjoined Q-structure space:

$1 - R^{a}_{bcd} = 2B^{a}_{bcd}$

$2 - R^{\hat{a}}_{bcd} = -4B_{[c|ab|d]}$ $3 - R^{\hat{a}}_{\hat{b}cd} = -2B^{b}_{acd}$

$4 - R^{a}_{\hat{b}\hat{c}\hat{d}} = -4B^{[c|ab|d]}$ $5 - R^{a}_{\hat{b}\hat{c}d} = -2B^{cab}_{d}$

$6 - R^{a}_{b\hat{c}\hat{d}} = 2B^{adc}_{b}$ $7 - R^{a}_{\hat{b}c\hat{d}} = 2B^{dab}_{c}$

$8 - R^{\hat{a}}_{b\hat{c}d} = 2B_{dab}{}^{c}$ $9 - R^{\hat{a}}_{\hat{b}\hat{c}\hat{d}} = B^{bcd}{}_{a}$

$10 - R^{\hat{a}}_{bc\hat{d}} = 2B_{cab}{}^{d}$ $11 - R^{a}_{bc\hat{d}} = 2B^{adh}B_{hbc} - 4B^{dah}B_{cbh} + A^{ad}_{bc}$

$12 - R^{a}_{b\hat{c}d} = 4B^{cah}B_{dbh} - A^{ac}_{bd} - 2B^{ach}B_{hbd}$

$13 - R^{\hat{a}}_{\hat{b}c\hat{d}} = 4B^{dbh}B_{cah} - A^{bd}_{ac} - 2B^{bdh}B_{hac}$ $14 - R^{\hat{a}}_{b\hat{c}\hat{d}} = 4B^{h}cdB_{hab}$

$15 - R^{\hat{a}}_{\hat{b}\hat{c}d} = 2B^{bch}B_{had} + A^{bc}_{ad} - 4B_{dah}B^{cbh}$

$16 - R^{a}_{\hat{b}cd} = 4B^{hab}B_{hdc}$

Definition 2.5[1]

The adjoined Q-structure of almost manifold has J-invariant Recci, if and only if, $r_{ab} = r^{\hat{a}}_{b} = 0$.

Main Results

Definition 3.1[10].

The projective manifold is defined as follows: it is a tensor of type (4,0) :

$P_{ijkl} = R_{ijkl} - \frac{1}{2n}[r_{ik}g_{jl} - r_{jk}g_{il}]$ (3.1)

where R_{ijkl}, r_{il} and g_{jk} are the Riemann tensor, Ricci tensor, and Riemann metric components, respectively. This tensor has features that are comparable to those of Riemann curvature,

$P_{ijkl} = -P_{jikl} = -P_{ijlk} = P_{klij}$.

Definition3.2 [8]

The Hermitian manifold is a manifold of class in the adjacent Q-structure space:

R_1 if and only if, $R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0$;

R_2 if and only if, $R_{abcd} = R_{\hat{a}bcd} = 0$;

R_3(RK-manifold) if and only if, $R_{\hat{a}bcd} = 0$;

Definition3.3

The adjacent Q-structure spàce, the Almost manifold is a class manifold.:

AR_1 if and only if, $P_{abcd} = P_{\hat{a}bcd} = P_{\hat{a}\hat{b}cd} = 0$;

AR_2 if and only if, $P_{abcd} = P_{\hat{a}bcd} = 0$;

AR_3 if and only if, $P_{\hat{a}bcd} = 0$;

Proposition 3.4 [3]

Let M be a random AH-manfiold, then AH-structure {J,g = < ., .>} is:
1. Almost Kahler structure if and only if $B_c^{ab} = B_{ab}^c = 0$, $B^{\{abc\}} = B_{\{abc\}} = 0$.
2. Kahler structure if and only if $B_c^{ab} = B_{ab}^c = 0$, $B^{abc} = B_{abc} = 0$.

Theorem 3.5

The compounds of the Almost Kahler projective tensor the following forms provide in the adjoined Q-structure:

Proof:

1) "Put $i = a, j = b, k = c$" , and "$l = d,$"

"$P_{abcd} = R_{abcd} - \frac{1}{2n}[r_{ac}g_{bd} - r_{bc}g_{ad}]$"

$P_{abcd} = 2B_{bcd}^a - \frac{1}{2n}(0) = 2B_{bcd}^a.$

2) "Put $i = \hat{a}, j = b, k = c$", and "$l = d$"

"$P_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2n}[r_{\hat{a}c}g_{bd} - r_{bc}g_{\hat{a}d}]$"

$P_{\hat{a}bcd} = -4B_{[c|ab|d]} - \frac{1}{2n}[-r_{bc}\delta_d^a] = -4B_{[c|ab|d]} + \frac{1}{2n}[r_{bc}\delta_d^a].$

3) "Put $i = a, j = \hat{b}, k = c,$ and $l = d$"

"$P_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{1}{2n}[r_{ac}g_{\hat{b}d} - r_{\hat{b}c}g_{ad}]$"

$P_{a\hat{b}cd} = -2B_{acd}^b - \frac{1}{2n}[-r_{ac}\delta_d^b] = -2B_{acd}^b + \frac{1}{2n}[r_{bc}\delta_d^a].$

4) "Put $i = a, j = b, k = \hat{c},$ and $l = d$"

"$P_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{1}{2n}[r_{a\hat{c}}g_{bd} - r_{b\hat{c}}g_{ad}]$"

$P_{ab\hat{c}d} = 2B_{dab}{}^{c} - \frac{1}{2n}(0) = 2B_{dab}{}^{c}.$

5) "Put $i = a, j = b, k = c,$" "and $l = \hat{d}$"

"$P_{abc\hat{d}} = R_{abc\hat{d}} - \frac{1}{2n}[r_{ac}g_{b\hat{d}} - r_{bc}g_{a\hat{d}}]$"

$P_{abc\hat{d}} = 2B\,{}_{cab}^{d} - \frac{1}{2n}[r_{ac}\delta_b^d - r_{bc}\delta_a^d].$

6) "Put $i = \hat{a}, j = \hat{b},$" $k = c,$ and $l = d$"

"$P_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{1}{2n}[r_{\hat{a}c}g_{\hat{b}d} - r_{\hat{b}c}g_{\hat{a}d}]$"

$P_{\hat{a}\hat{b}cd} = 4B^{hab}B_{hdc} - \frac{1}{2n}[r_c^a\delta_d^b - r_c^b\delta_d^a].$

7) "Put $i = \hat{a}, j = b, k = \hat{c},$ and $l = d$"

"$P_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{1}{2n}[r_{\hat{a}\hat{c}}g_{bd} - r_{b\hat{c}}g_{\hat{a}d}]$"

$P_{\hat{a}b\hat{c}d} = 4B^{cah}B_{dbh} - A_{bd}^{ac} - 2B^{ach}B_{hbd} - \frac{1}{2n}[0 - r_b^c\delta_d^a]$

$P_{\hat{a}b\hat{c}d} = 4B^{cah}B_{dbh} - A_{bd}^{ac} - 2B^{ach}B_{hbd} + \frac{1}{2n}[r_b^c\delta_d^a].$

8) "Put $i = a, j = \hat{b}, k = c,$ and $l = \hat{d}$"

"$P_{a\hat{b}c\hat{d}} = R_{a\hat{b}c\hat{d}} - \frac{1}{2n}[r_{ac}g_{\hat{b}\hat{d}} - r_{\hat{b}c}g_{a\hat{d}}]$"

$P_{a\hat{b}cd} = 4B^{dbh}B_{cah} - A_{ac}^{bd} - 2B^{bdh}B_{hac} - \frac{1}{2n}[0 - r_c^b\delta_a^d]$

$P_{a\hat{b}c\hat{d}} = 4B^{dbh}B_{cah} - A_{ac}^{bd} - 2B^{bdh}B_{hac} + \frac{1}{2n}[r_b^c\delta_d^a].$

Theorem3.6

Let the manifold M of Projective tensor of (AK-manifold),and M has J-invariant Recci tensor, Then M is a holomorphic sectional.

Proof:
Assume M is (AK-manifold), then
By theorem(3.5), we have
$4B^{cah}B_{dbh} - A^{ac}_{bd} - 2B^{ach}B_{hbd} + \frac{1}{2n}[r^c_b\delta^a_d]$
$4(\frac{1}{2}(B^{cah}B_{dbh}+B^{ach}B_{dbh}) - A^{ac}_{bd} - 2(\frac{1}{2}(B^{ach}B_{hbd}+(B^{cah}B_{hbd}) + \frac{1}{2n}[r^c_b\delta^a_d]$
$4(\frac{1}{2}(B^{cah}B_{dbh}-B^{cah}B_{dbh}) - A^{ac}_{bd} - (B^{ach}B_{hbd}-(B^{ach}B_{hbd}) + \frac{1}{2n}[r^c_b\delta^a_d] = 0$
Since M has Ĵ-invariant Rɨcci, then
$\frac{1}{2n}[r^c_b\delta^a_d] = 0$
$-A^{ac}_{bd}=0 \Leftrightarrow A^{ac}_{bd}=0$
Hence, M is a flat holomorphic sectional.

Theorem3.7
Assume M is a manifold of Almost Kahlar ,"then M has" J-invaiant "Ricci tensor",
"Proof:"
"Suppose that M is" (AK-"manifold), then"
By theorem(3.5), we have
$4B^{cah}B_{dbh} - A^{ac}_{bd} - 2B^{ach}B_{hbd} + \frac{1}{2n}[r^c_b\delta^a_d]$
By theorem(3.6), we have
$A^{ac}_{bd}=0$
Hence
$4B^{cah}B_{dbh} - 2B^{ach}B_{hbd} + \frac{1}{2n}[r^c_b\delta^a_d] = 0$
contracting through the index (a,h), we deduce
$\frac{1}{2n}[r^c_b\delta^a_d] = 0$
$\frac{1}{2(2n)}(r^c_b\delta^a_d - r^a_d\delta^c_b) = 0$
contracting through the index (a,d), we have
$\frac{1}{4n}(r^c_b\delta^a_a - r^a_a\delta^c_b) = 0$
$\frac{1}{4n}(nr^c_b - r^a_a\delta^c_b) = 0$
Antisymmetrizing and symmetries by the indices (a,c) we obtain
$\frac{1}{4}(r^c_b) = 0 \Rightarrow r^c_b = 0$
Therefore M has a J-invariant Ricci tensor.

Theorem3.8
Let M be a manifold of Almost Kahlar, Then M is Kahler
Proof:
Assume M is (AK-manifold), then
By theorem(3.5), we have

$4B^{hab}B_{hdc} - \frac{1}{2n}[r_c^a\delta_d^b - r_c^b\delta_d^a]$

Symmetries and Antisymmetrizing "by the indices" "(c,b)" and (c,a) "we obtain"

$4B^{hab}B_{hdc} = 0$

contracting through "the indices (a,d) , (b,c)", we have:

$4B^{hab}B_{hab}$⇒"$B^{hab}B_{hab} = 0$"

$B^{hab}\overline{B}^{hab} = 0 \Rightarrow \sum_{a,b,h}$"$\left|B^{hab}\right|^2 = 0 \Leftrightarrow B^{hab} = 0$ "

By Proposition (3,4) we have

Hence M is Kahler manifold.

Theorem 3.9

Let M be a manifold of Almost Kahlar, then M is a Locally conformal Kahler manifold (LCK-manifold).

Proof:

Suppose that M is (AK-manifold), then

By theorem (3.5), we have

$4B^{hab}B_{hdc} - \frac{1}{2n}[r_c^a\delta_d^b - r_c^b\delta_d^a]$

Symmetries and Anti symmetrizing by the indices (c, b) and (c, a) we obtain

$4B^{hab}B_{hdc} = 0$

contracting through the indexes (d,a) and (c,b), we get:

$4B^{hdc}B_{hdc} \Rightarrow B^{hdc}B_{hdc} = 0$

Hence

$B^{hdc}B_{hdc} = 0 \Rightarrow \overline{B}_{hdc}B_{hdc} = 0 \Rightarrow \sum_{h,d,c}|B_{hdc}|^2 = 0 \Leftrightarrow B_{hdc} = 0$

Hence M is a Locally conformal Kahler.

Conclusions

1- Studied some components Projective of Almost Kahler**.**
2- To prove that the manifold M is a flat holomorphic sectional tensor of projective Almost Kahlar manifold.
3- To prove that the manifold M has J-invariant Ricci tensor.
4- proved that (AK-manifold) M is kahler manifold.
5- There exists relation between Almost Kahlar manifold (AK-manifold) and Lacally confermal Kahler manifold (LCK-manifeld).

References

1. Shihab, A., & Kirichenko, F.V. Rustanov , A. (2011). On geometry of the tensor of conharmonic curvature of almost Hermitian. Mathematics. *Notes, Moscow* Vol.90 , No-1 ,pp. 87-103 ,.
2. Ali A. Shihab & Ali K. Ali, (2022). M-projective Curvature Tensor of Nearly Kahler Manifold. Eurasian Journal of Physics, *Chemistry and Mathematics, Vol*. 8, PP. (41-46).
3. Ali A. Shihab & Alham M. Mohammed ,(2021). N Generalized Conharmonic Curvature Tensor of the Locally Conformal Kahler Manifold, *Turkish Journal of Computer and Mathematics Education, Vol*. 12, No.11

4. Gray A. (1965). Minimal varieties and almost Hermitian submanifolds. *Michigan Math*. J.,12, , P.273-279.
5. Gray A.(1970). Nearly K hler manifold, J. diff. geom. (V.4, p. 283-309).
6. Jawad J. M.(2004). Almost Kahler manifold of class R_1., M.Sc. thesis, University of Basrah, College of Science,.
7. Jawarneh M., Samui .(2017). Projective curvature tensor on (k,μ)-contact space forms, *Journal of Pure and Applied Mathematics*, (V. 113, No.3, P.425-439).
8. Kartan A.(1960). Riemannian geometry in the orthonormal frame, Moscow State University, Moscow.
9. Kirichenko V.F.(1976). K – spaces of constant type, *Seper. Math.J*. (Vol. T. 17, No. 2, pp. 282-289).
10. Kirichenko VF.(1975). New results of K–spaces theory. Ph. D. thesis, Moscow state University.
11. Kirichenko V. F.(1995). Theory of Lie Groups, Tver state University.
12. Kobayashi S, Nomizu K. Foundations of differential geometry. John Wily and Sons.1963;1.
13. Koto S. Some theorems on almost Kählerian spaces, J.math. Soc. Japan, 12,1960, P.422-433.
14. Rachevski PK. Riemmanian geometry and tensor analysis. M. Nauka; 1964.
15. Yildirim, U., Atceken, M. and Dirik, S. (2019). Pseudo Projective Curvature Tensor Satisfying Some Properties On A Normal Paracontact Metric Manifold", *Commun. Fac. Sci. Univ. Ank. Ser.A1 Math. Stat.*, V. 68, No.1, pp. 997-1016.

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Samarra Journal of Pure and Applied Science

SJPAS
مجلة سامراء للعلوم الصرفة والتطبيقية

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

منطوي كوهلر التقريبي للتنزر الاسقاطي

عبد الهادي احمد عبد
مديرية تربية صلاح الدين

معلومات البحث:

تأريخ الاستلام: 2024/01/25
تاريخ التعديل: 2024/02/24
تأريخ القبـــول: 2024/02/28
تاريخ الـــنشر: 2024/03/30

الكلمات المفتاحية:

تنزر إسقاطي، منطوي كوهلر التقريبي، المنطوي الهرميتي التقريبي

معلومات المؤلف

الايميل:
الموبايل:+

الخلاصة:

العمل من هذا البحث هو تحليل الخصائص الهندسية للتنزر التوافقي الاسقاطي لمنطوي كوهلر التقريبي. نحن نستخدم خصائص تسطيح تنزر الانحناء الإسقاطي لتحديد مركبات منطوي كوهلر التقريبي. تم ايجاد بعض المركبات لهذا المنطوي، كما تم برهان ان المنطوي هو تنزر مقطعي مسطح، وتم اثبات ان هذا المنطوي يمتلك تنزر ريجي الثابت، وايضا تم اثبات ان هذا المنطوي يمثل منطوي كوهلر, واخيرا تم ايجاد علاقة بين هذا المنطوي وبين منطوي (Locally conformal Kahler).